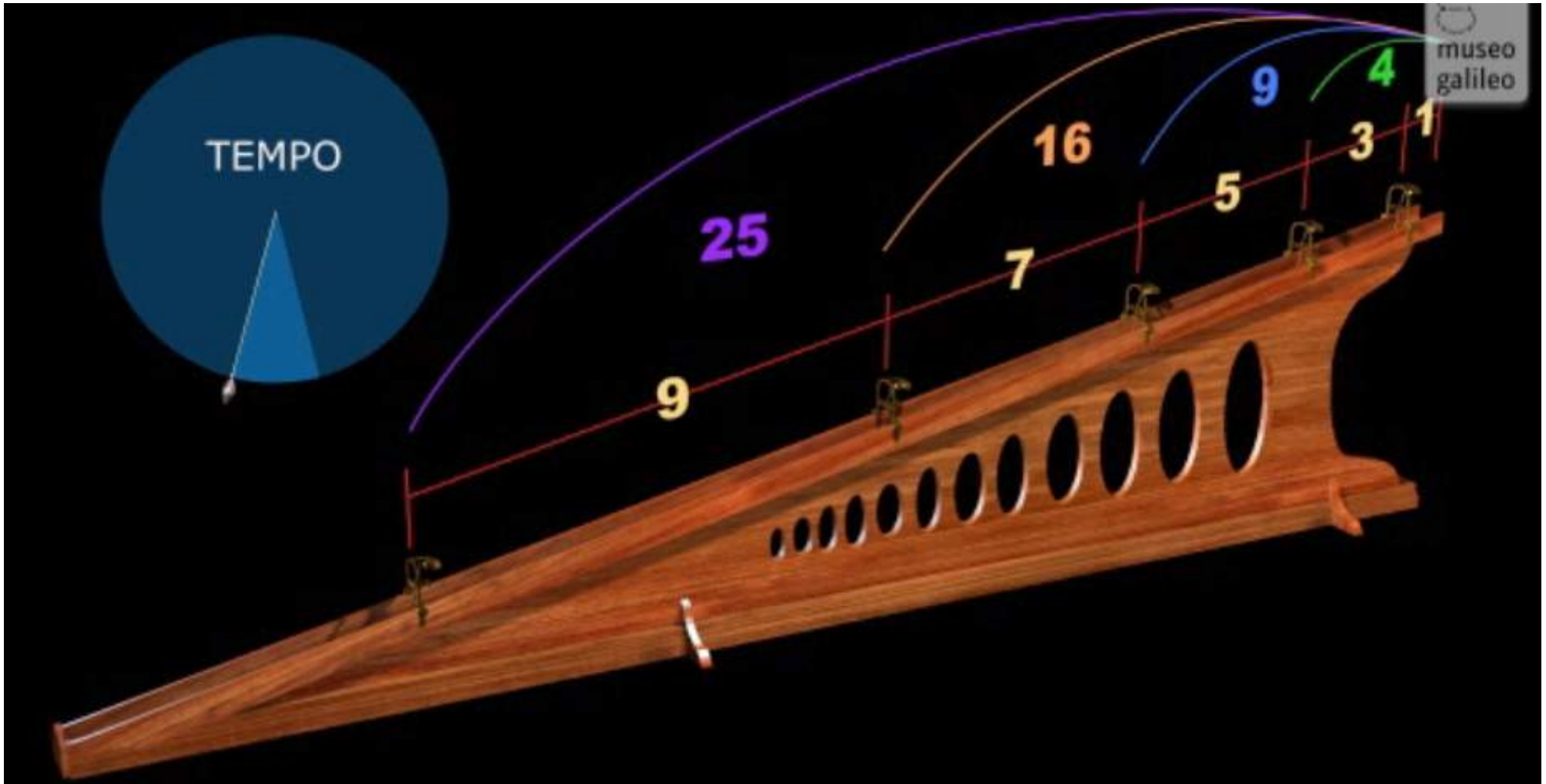
Esempio 5 Il primo esperimento scientifico moderno (1604)

DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuoue scienze
Attenenti alla
MECANICA & I MOVIMENTI LOCALI,
del Signor
GALILEO GALILEI LINCEO,
Filosofo e Matematico primario del Serenissimo
Grand Duca di Toscana.
Con vna Appendice del centro di grauità d'alcuni Solidi.

IN LEIDA,
Appresso gli Elsevirii. M. D. C. XXXVIII.

Sensate esperienze
e dimostrazioni matematiche

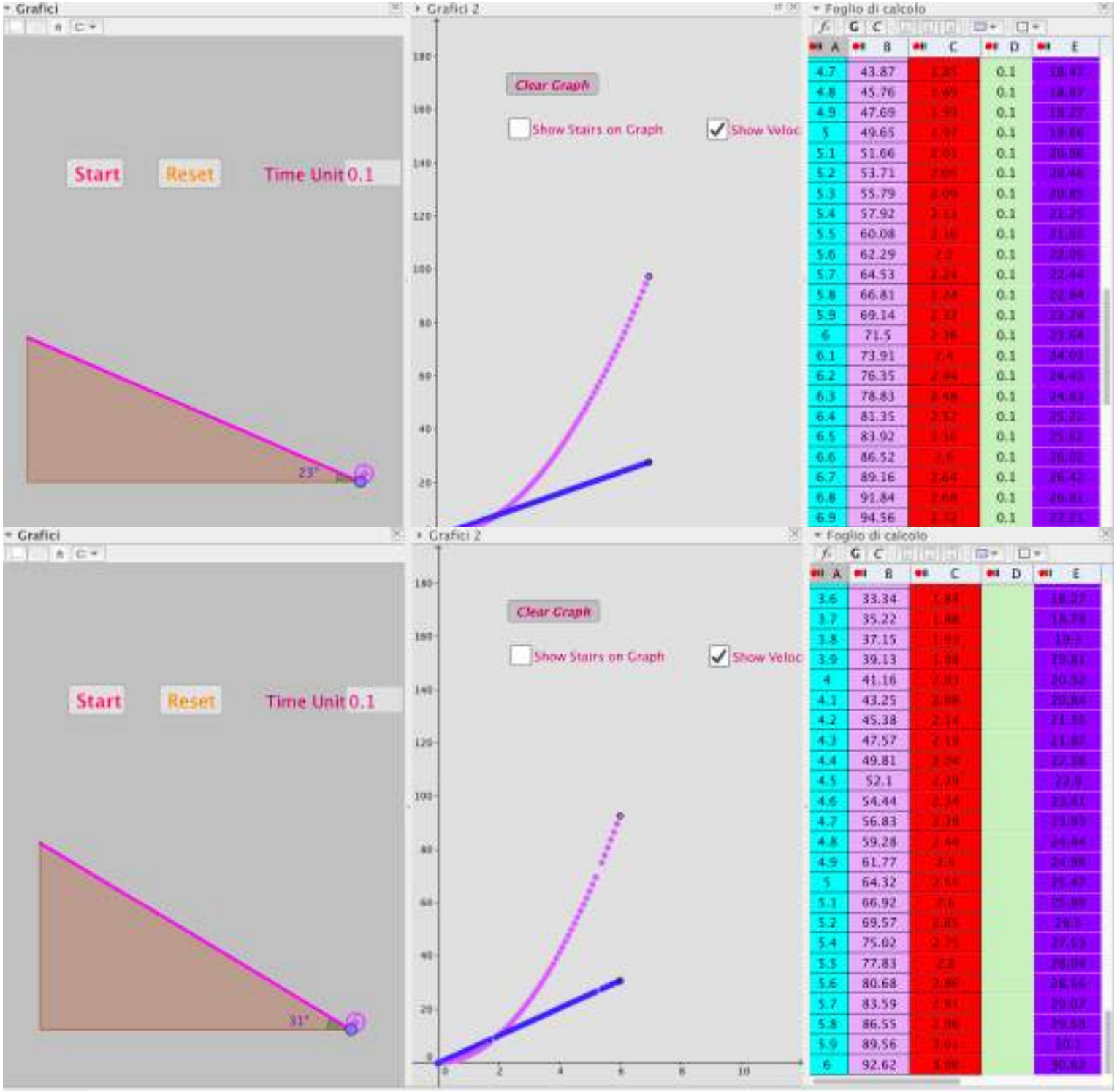
http://catalogo.museogalileo.it/multimedia/PianoInclinato.html

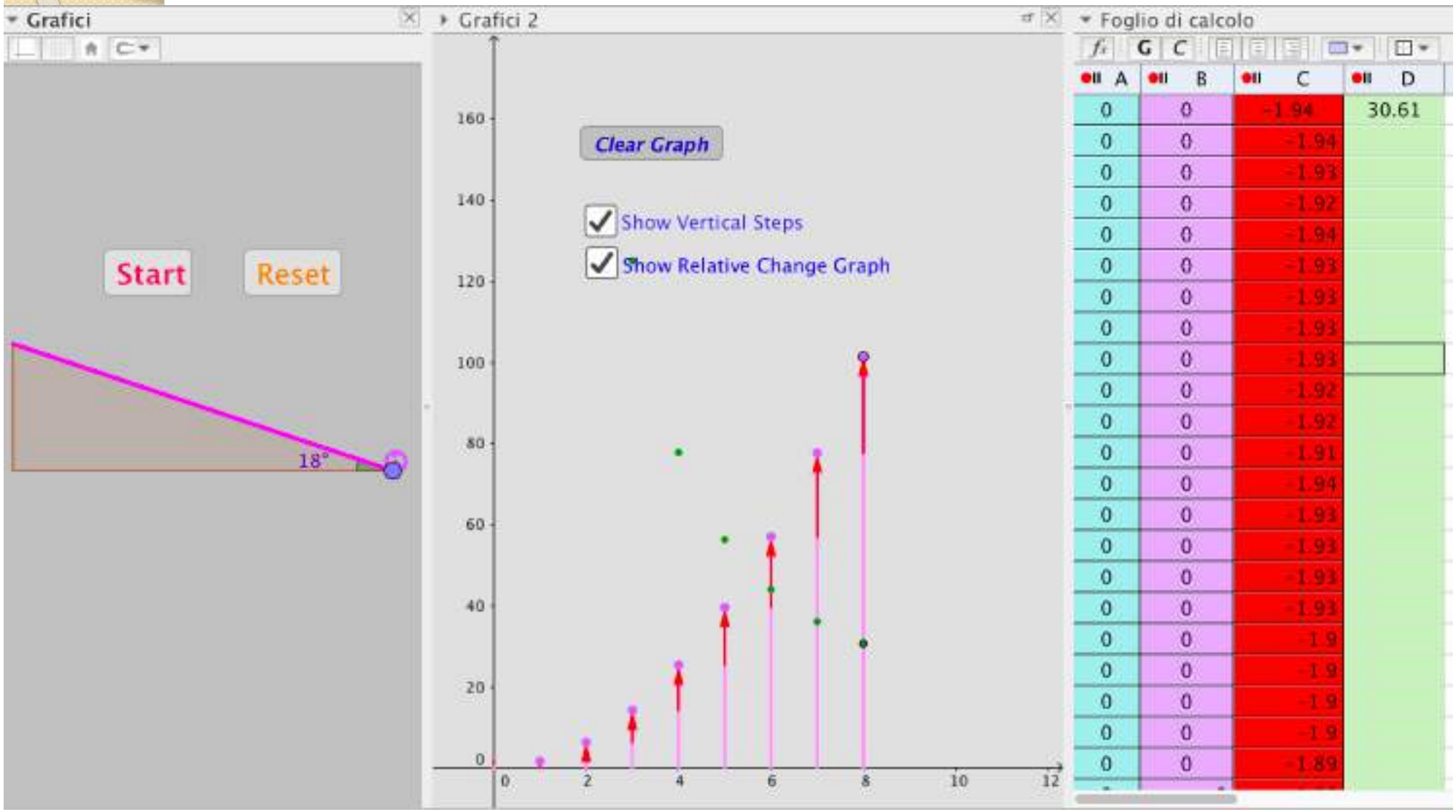


Si conclude che:

- alla base del piano inclinato il corpo ha sempre la stessa velocità indipendentemente dall'inclinazione e dalla lunghezza del piano.
- in altre parole, la velocità raggiunta alla base del piano inclinato dipende dalla quota alla quale il corpo si trova inizialmente.

Quindi anche in caduta libera avrà la stessa velocità.


Grafici
Start
Reset
Time Unit 0.1
23°
Grafici 2
Clear Graph
Show Stairs on Graph
Show Veloc
Foglio di calcolo
A
B
C
D
E
4.7 43.87 0.1
4.8 45.76 0.1
4.9 47.69 0.1
5 49.65 0.1
5.1 51.66 0.1
5.2 53.71 0.1
5.3 55.79 0.1
5.4 57.92 0.1
5.5 60.08 0.1
5.6 62.29 0.1
5.7 64.53 0.1
5.8 66.81 0.1
5.9 69.14 0.1
6 71.5 0.1
6.1 73.91 0.1
6.2 76.35 0.1
6.3 78.83 0.1
6.4 81.35 0.1
6.5 83.92 0.1
6.6 86.52 0.1
6.7 89.16 0.1
6.8 91.84 0.1
6.9 94.56 0.1
Grafici
Start
Reset
Time Unit 0.1
31°
Grafici 2
Clear Graph
Show Stairs on Graph
Show Veloc
Foglio di calcolo
A
B
C
D
E
3.6 33.34
3.7 35.22
3.8 37.15
3.9 39.13
4 41.16
4.1 43.25
4.2 45.38
4.3 47.57
4.4 49.81
4.5 52.1
4.6 54.44
4.7 56.83
4.8 59.28
4.9 61.77
5 64.32
5.1 66.92
5.2 69.57
5.4 75.02
5.5 77.83
5.6 80.68
5.7 83.59
5.8 86.55
5.9 89.56
6 92.62


Grafici
Start
Reset
18°
Grafici 2
Clear Graph
Show Vertical Steps
Show Relative Change Graph
160
140
120
100
80
60
40
20
0
0
2
4
6
8
10
12
Foglio di calcolo
A
B
C
D
0 0 -1.94 30.61
0 0 -1.94
0 0 -1.93
0 0 -1.92
0 0 -1.94
0 0 -1.93
0 0 -1.93
0 0 -1.93
0 0 -1.93
0 0 -1.92
0 0 -1.92
0 0 -1.91
0 0 -1.94
0 0 -1.93
0 0 -1.93
0 0 -1.93
0 0 -1.93
0 0 -1.9
0 0 -1.9
0 0 -1.9
0 0 -1.9
0 0 -1.89

Apprendistato all'interpretazione dei grafici di funzione

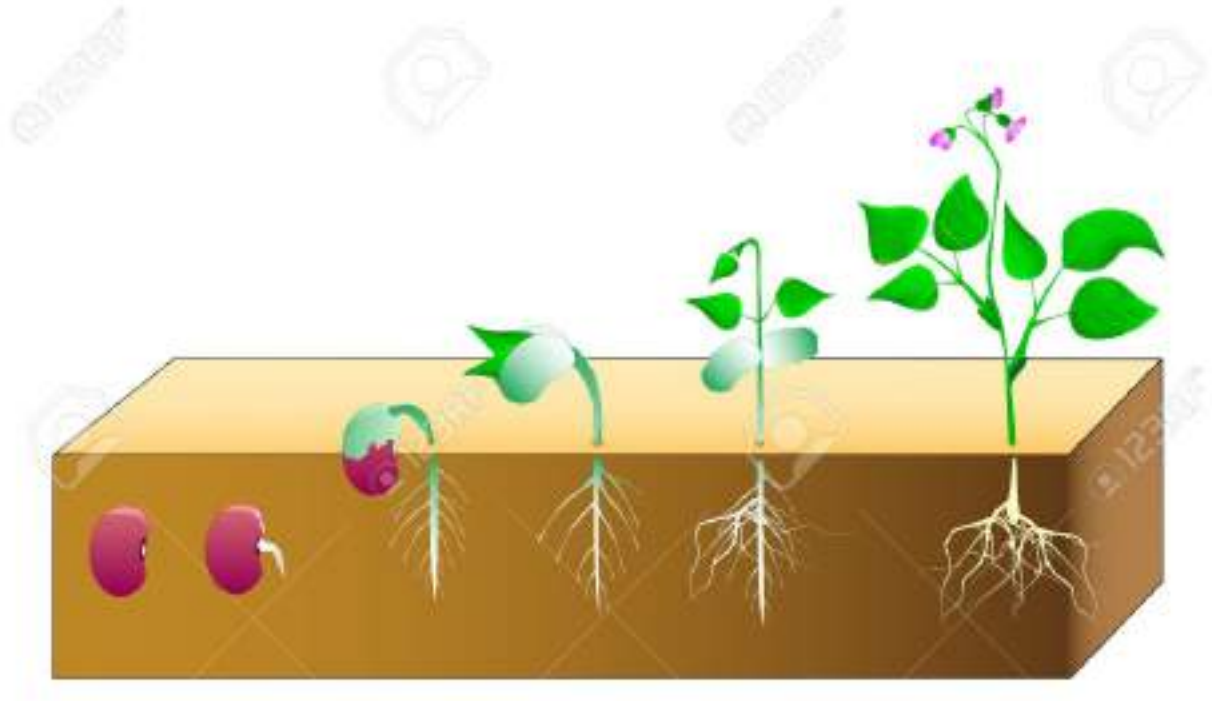
Occorre che gli allievi siano introdotti a un apprendistato nell'interpretazione dei grafici in vari campi di esperienza in cui si esperiscano significativi fenomeni di **cambiamento**:

- Movimento
- Crescita (decrescita) in situazioni varie:
  - Piante
  - Temperatura
  - Prezzi
  - Persone
  - …

2

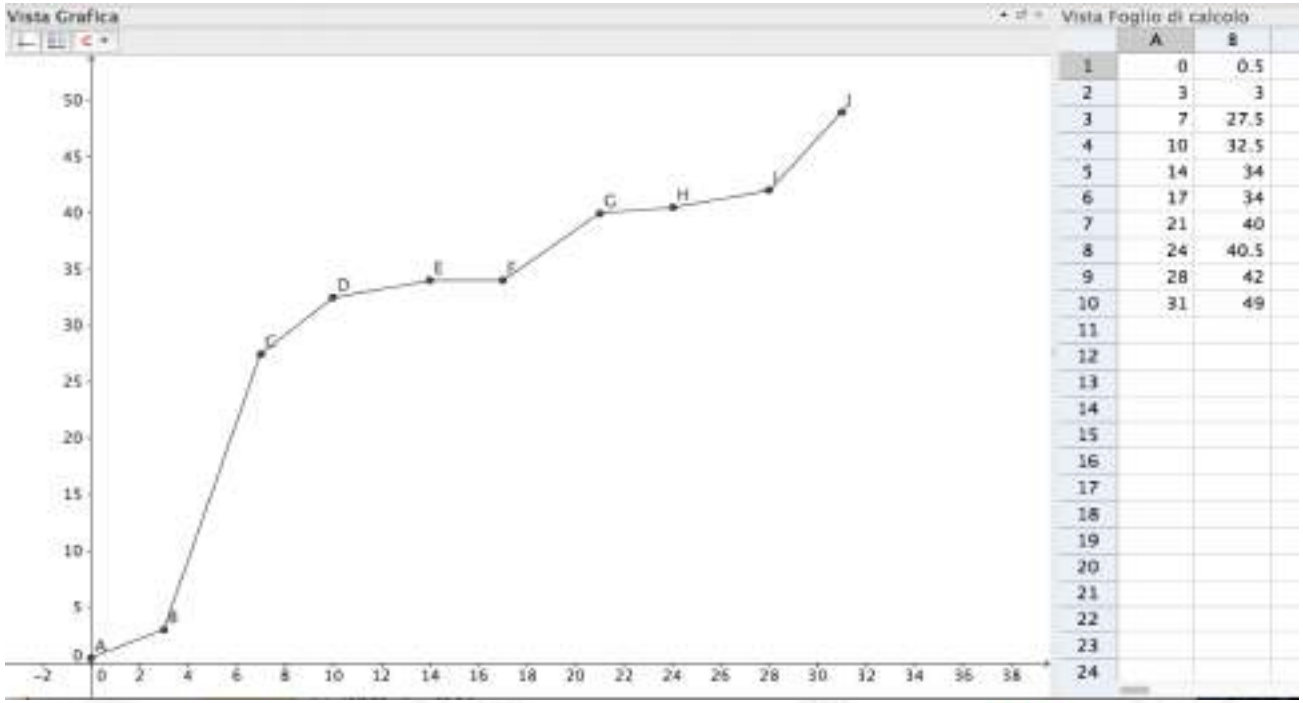
Esempio 6: crescita delle piante di fagioli

	sabbia	t.
lun 31/1/00	0,5	
gio 3/2/00	3	
lun 7/2/00	27,5	
gio 10/2/00	32,5	
lun 14/2/00	34	
gio 17/2/00	34	
lun 21/2/00	40	
gio 24/2/00	40,5	
lun 28/2/00	42	
gio 2/3/00	49	

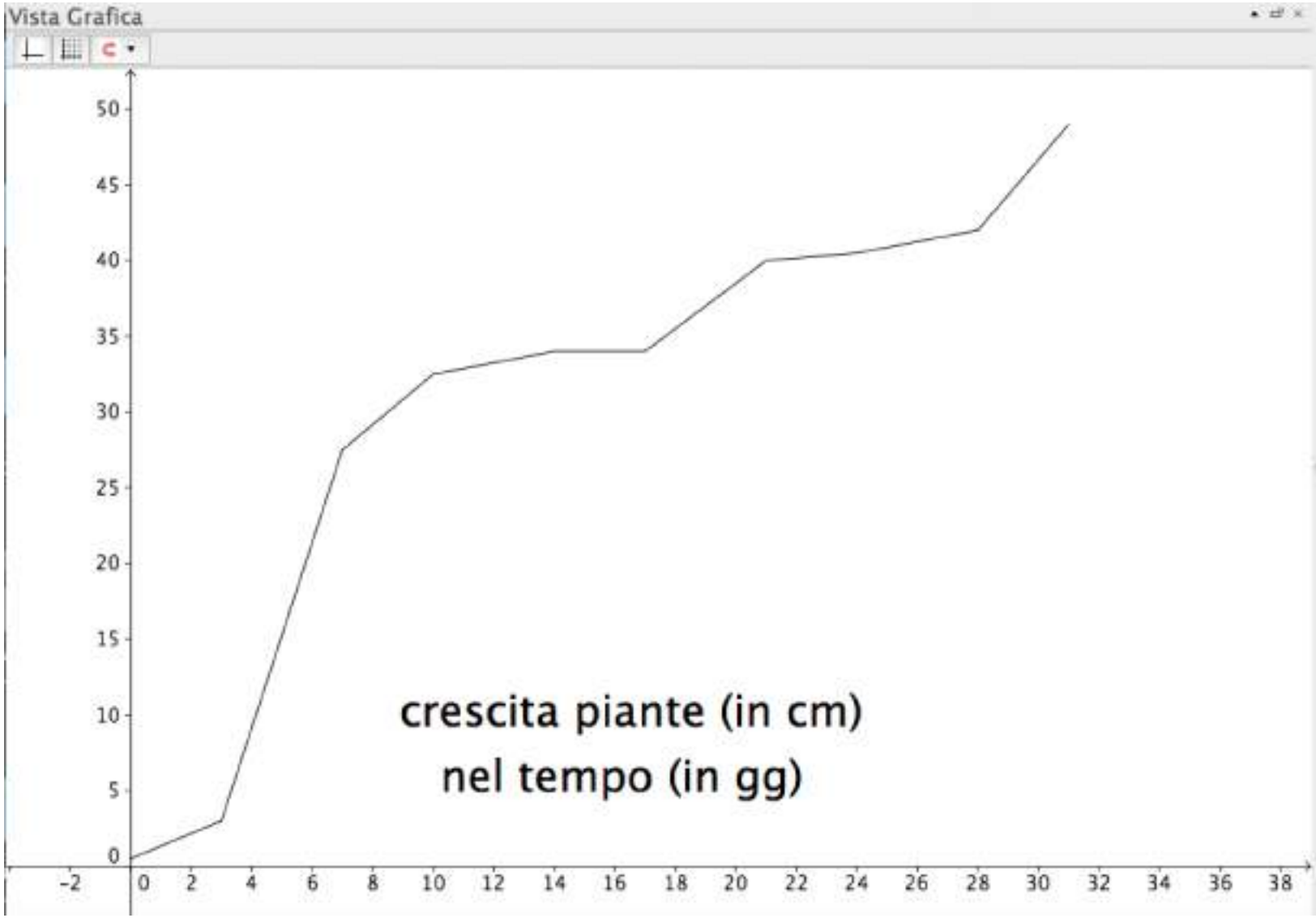


Crescita delle piante di fagioli

	sabbia	t.
lun 31/1/00	0,5	
gio 3/2/00	3	
lun 7/2/00	27,5	
gio 10/2/00	32,5	
lun 14/2/00	34	
gio 17/2/00	34	
lun 21/2/00	40	
gio 24/2/00	40,5	
lun 28/2/00	42	
gio 2/3/00	49	



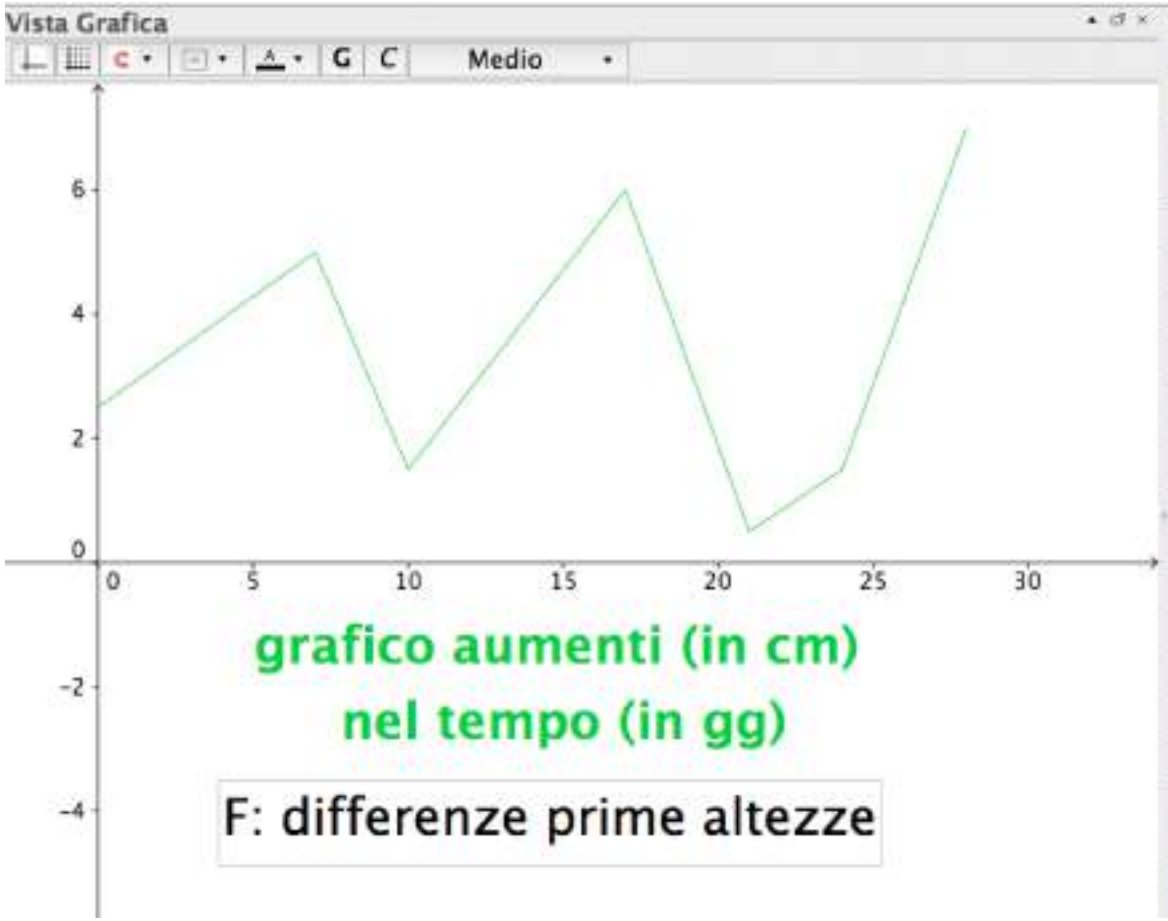
Crescita delle piante di fagioli



Vista Foglio di calcolo

	A	B
1	0	0.5
2	3	3
3	7	27.5
4	10	32.5
5	14	34
6	17	34
7	21	40
8	24	40.5
9	28	42
10	31	49
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		

Crescita delle piante di fagioli



Vista Foglio di calcolo

	A	B	C	D	E	F
1	0	0.5	2.5	3	0	2.5
2	3	3	24.5	4	3	24.5
3	7	27.5	5	3	7	5
4	10	32.5	1.5	4	10	1.5
5	14	34	0	3	14	0
6	17	34	6	4	17	6
7	21	40	0.5	3	21	0.5
8	24	40.5	1.5	4	24	1.5
9	28	42	7	3	28	7
10	31	49				
11		0				
12						
13						
14						
15						
16						
17						
18						
19						
20						

Esempio 7
Temperature e umidità

Si veda a livello introduttivo: Il significato di grado sul termometro
(Mat. 2001, Argomentare)

7:30

8:00

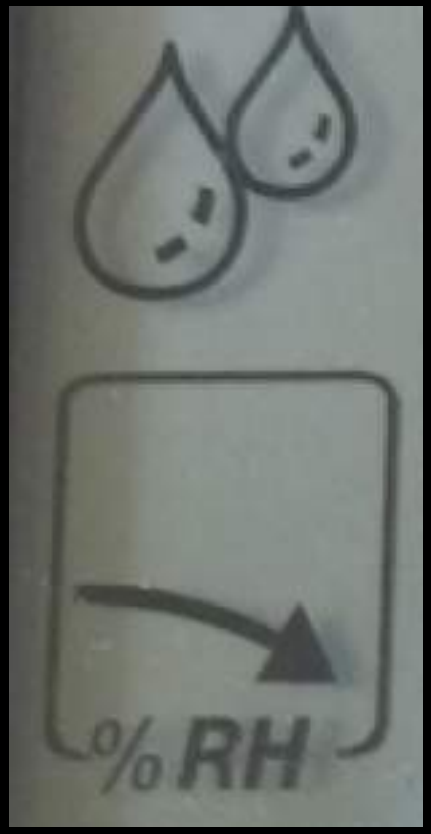
8:30

9:00

10:00

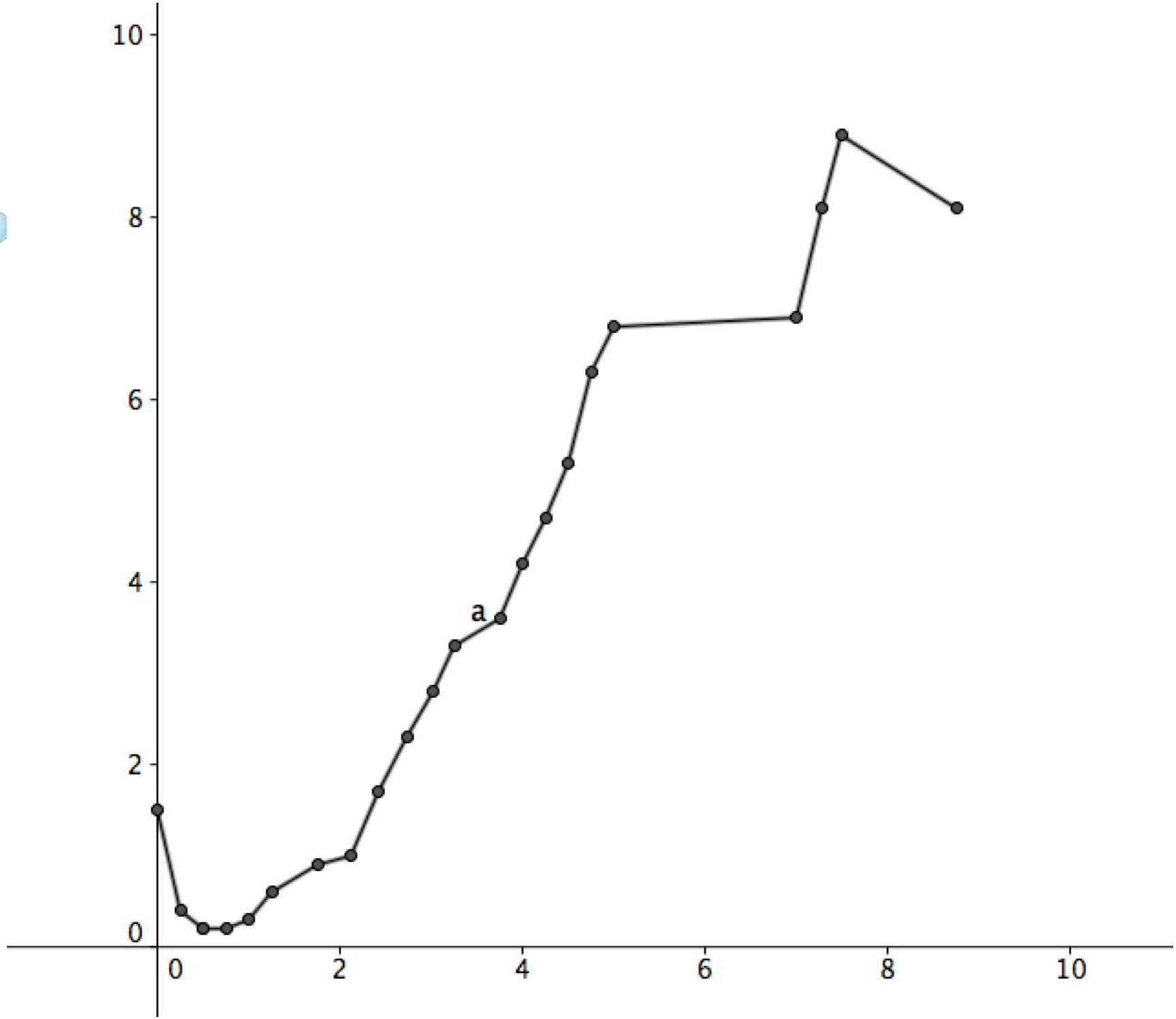
11:00

Che cosa rappresentano le frecce?

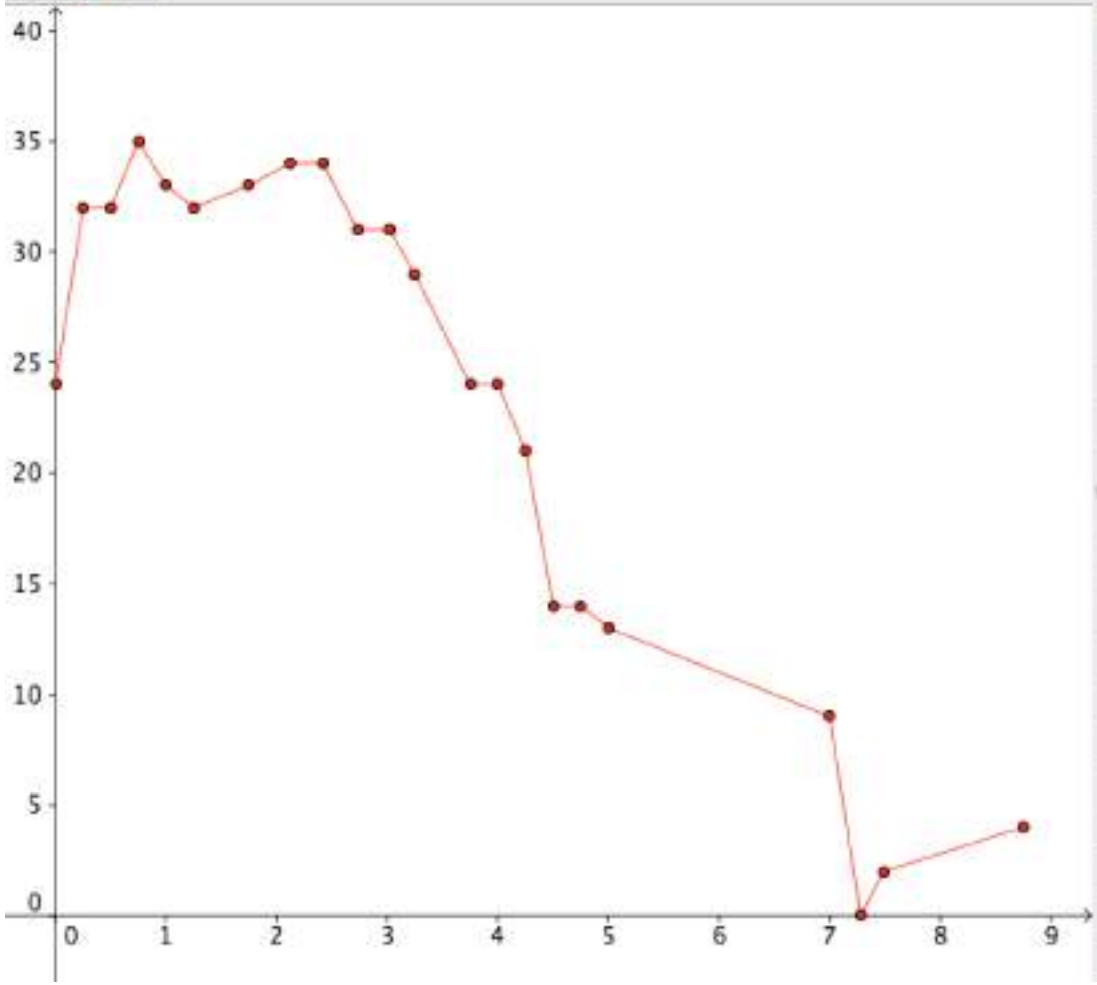


Temperature e umidità

ORA	Gradi Celsius C°	Umid. Relativa %	Indice Humidex	ORA	Gradi Celsius C°	Umid. Relativa %	Indice Humidex
7:15	18,5	74		7:15	21,3	64	
7:30	17,4	82		7:30	19,1	71	
7:45	17,2	82		7:45	18,4	77	
8:00	17,2	85		8:00	18,3	80	
8:15	17,3	83		8:15	18,6	81	
8:30	17,6	82		8:40	19,2	81	
9:00	17,9	83		9:05	20,2	82	
9:22	18,0	84		9:15	20,6	81	27
9:40	18,7	84		9:30	21,3	78	27
9:58	19,3	81		9:45	22,2	72	27
10:16	19,8	81		10:37	22,8	70	28
10:30	20,3	79		10:00	24,8	62	30*
11:00	20,6	74	26	11:10	26,4	54	31*
11:15	21,2	74	26	11:45	26,3	55	31*
11.30	21,7	71	27	16:45	27,7	38	28,5
11.45	22,3	64	26				
12:00	23,3	64	28,5				
12:15	23,8	63	28,5				
14:15	23,9	59	28				
14:32	25,1	50	30*				
14:44	25,9	52	30*				
18:00	25,1	54	28				


10
8
6
4
2
0
a
0
2
4
6
8
10

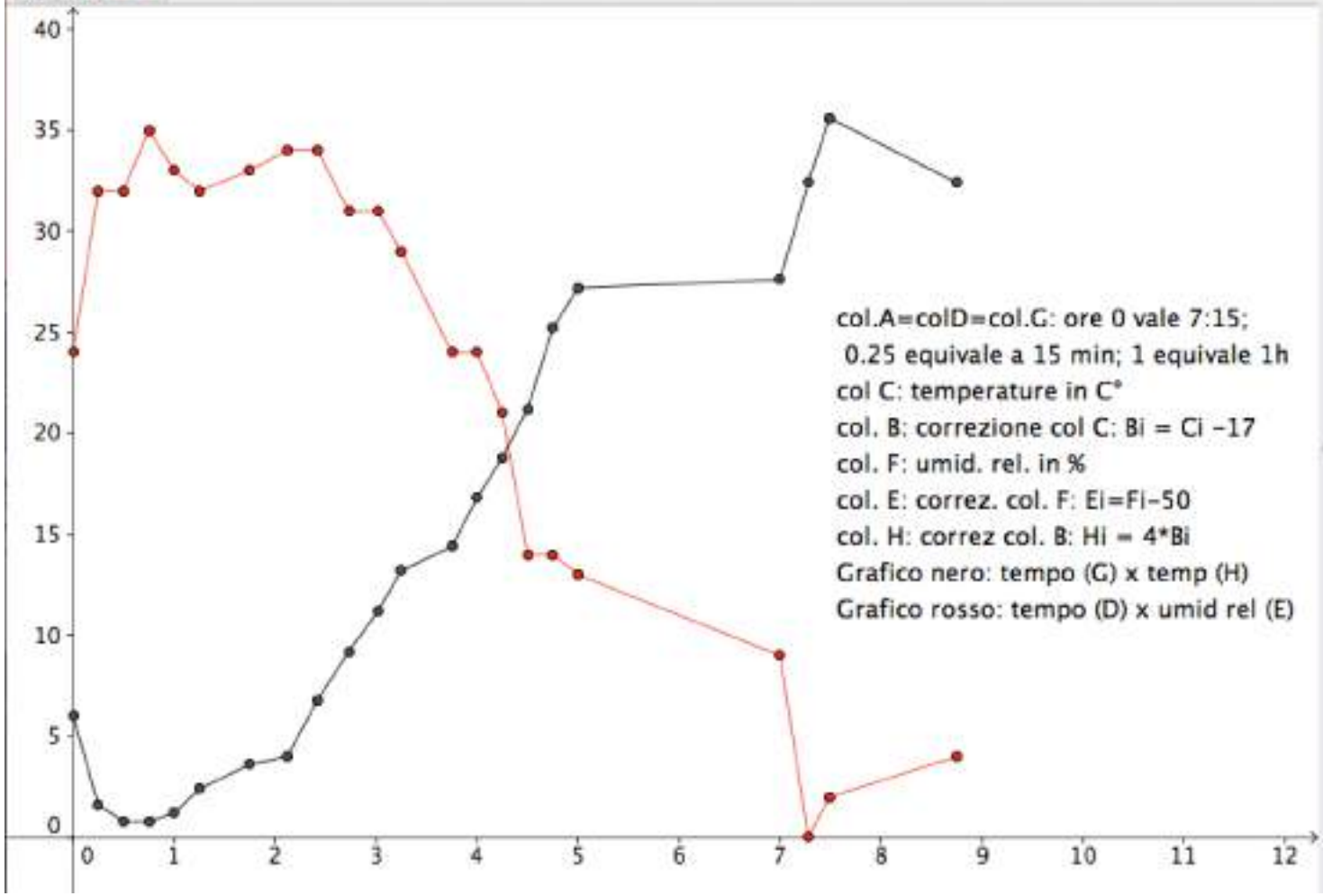
ista Grafica



Vista Foglio di calcolo

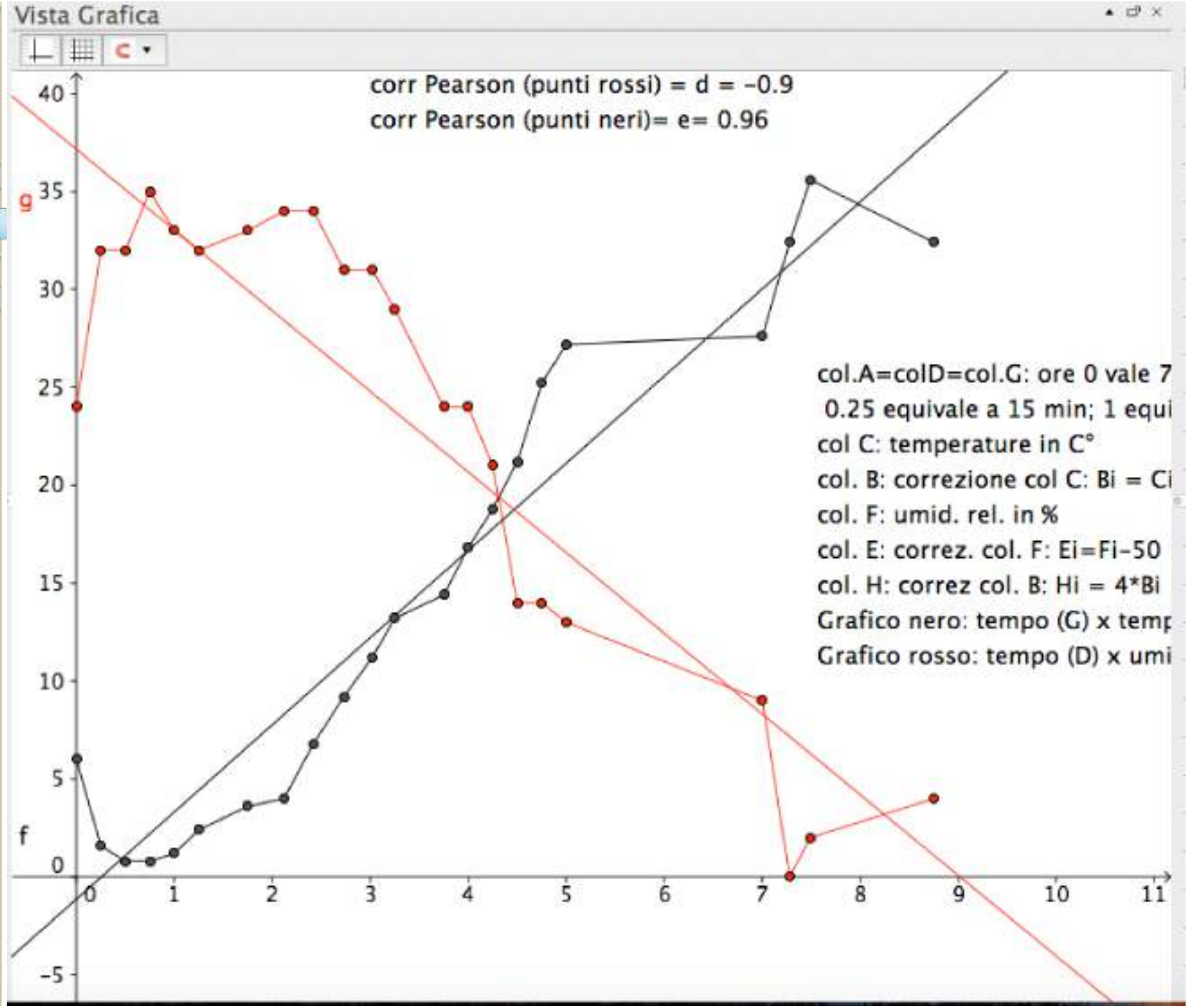
	A	B	C	D	E	F
1	0	1.5	18.5	0	24	74
2	0.25	0.4	17.4	0.25	32	82
3	0.5	0.2	17.2	0.5	32	82
4	0.75	0.2	17.2	0.75	35	85
5	1	0.3	17.3	1	33	83
6	1.25	0.6	17.6	1.25	32	82
7	1.75	0.9	17.9	1.75	33	83
8	2.12	1	18	2.12	34	84
9	2.42	1.7	18.7	2.42	34	84
10	2.73	2.3	19.3	2.73	31	81
11	3.02	2.8	19.8	3.02	31	81
12	3.25	3.3	20.3	3.25	29	79
13	3.75	3.6	20.6	3.75	24	74
14	4	4.2	21.2	4	24	74
15	4.25	4.7	21.7	4.25	21	71
16	4.5	5.3	22.3	4.5	14	64
17	4.75	6.3	23.3	4.75	14	64
18	5	6.8	23.8	5	13	63
19	7	6.9	23.9	7	9	59
20	7.28	8.1	25.1	7.28	0	50
21	7.49	8.9	25.9	7.49	2	52
22	8.75	8.1	25.1	8.75	4	54
23						

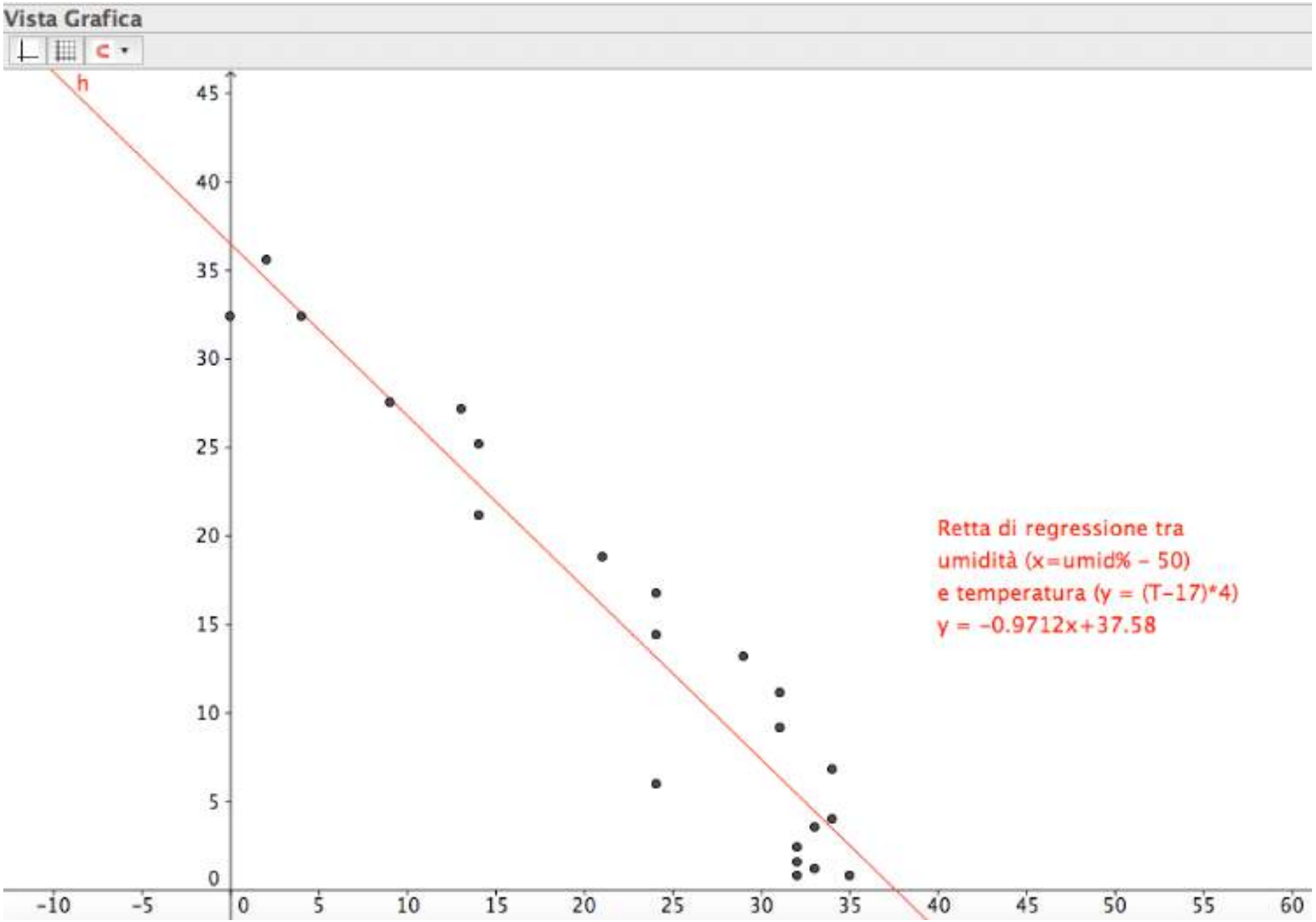
Vista Grafica



Vista Foglio di calcolo

	A	B	C
1	0	1.5	18.5
2	0.25	0.4	17.4
3	0.5	0.2	17.2
4	0.75	0.2	17.2
5	1	0.3	17.3
6	1.25	0.6	17.6
7	1.75	0.9	17.9
8	2.12	1	18
9	2.42	1.7	18.7
10	2.73	2.3	19.3
11	3.02	2.8	19.8
12	3.25	3.3	20.3
13	3.75	3.6	20.6
14	4	4.2	21.2
15	4.25	4.7	21.7
16	4.5	5.3	22.3
17	4.75	6.3	23.3
18	5	6.8	23.8
19	7	6.9	23.9
20	7.28	8.1	25.1
21	7.49	8.9	25.9
22	8.75	8.1	25.1
23			


Vista Grafica
corr Pearson (punti rossi) = d = -0.9
corr Pearson (punti neri)= e= 0.96
col.A=colD=col.G: ore 0 vale 7
0.25 equivale a 15 min; 1 equi
col C: temperature in C°
col. B: correzione col C: Bi = Ci
col. F: umid. rel. in %
col. E: correz. col. F: Ei=Fi-50
col. H: correz col. B: Hi = 4*Bi
Grafico nero: tempo (G) x temp
Grafico rosso: tempo (D) x umi
g
f
40
35
30
25
20
15
10
5
0
-5
0
1
2
3
4
5
6
7
8
9
10
11


Vista Grafica
h
Retta di regressione tra
umidità (x=umid% - 50)
e temperatura (y = (T-17)*4)
y = -0.9712x+37.58
45
40
35
30
25
20
15
10
5
0
-10
-5
0
5
10
15
20
25
30
35
40
45
50
55
60

Approfondiamo il cambiamento

Πάντα ρει

I processi di cambiamento: una **radice cognitiva** (D. Tall) per la matematica e la scienza

Il correlativo cognitivo del **cambiamento** è l'attenzione a ciò che cambia e come cambia e a ciò che rimane invariante in una situazione.

Il correlativo matematico del cambiamento è l'attenzione non solo ai valori quantitativi ma anche e soprattutto alle loro **differenze** e al modo di rappresentarle e manipolarle per ragionarci.

→ **LE DIFFERENZE FINITE**:

a) Uno strumento potente che permette di preparare il calcolo differenziale fin dai primi anni.
b) Uno strumento facilmente implementabile con i software didattici.

Differenze: una misura del cambiamento (quadrati)

a	$4(a-2)$	a	a^2	a	4	ΔB	ΔD
A	B	C	D	E	F	G	H
2	0	2	0	2	4	4	1
3	4	3	1	3	4	4	3
4	8	4	4	4	4	4	5
5	12	5	9	5	4	4	7
6	16	6	16	6	4	4	9
7	20	7	25	7	4	4	11
8	24	8	36	8	4	4	13
9	28	9	49	9	4	4	15
10	32	10	64	10	4	4	17
11	36	11	81	11	4	4	19
12	40	12	100	12	4	4	21
13	44	13	121	13	4	4	23
14	48	14	144	14	4	4	25
15	52	15	169	15	4	4	27
16	56	16	196	16	4	4	29
17	60	17	225	17	4	4	31

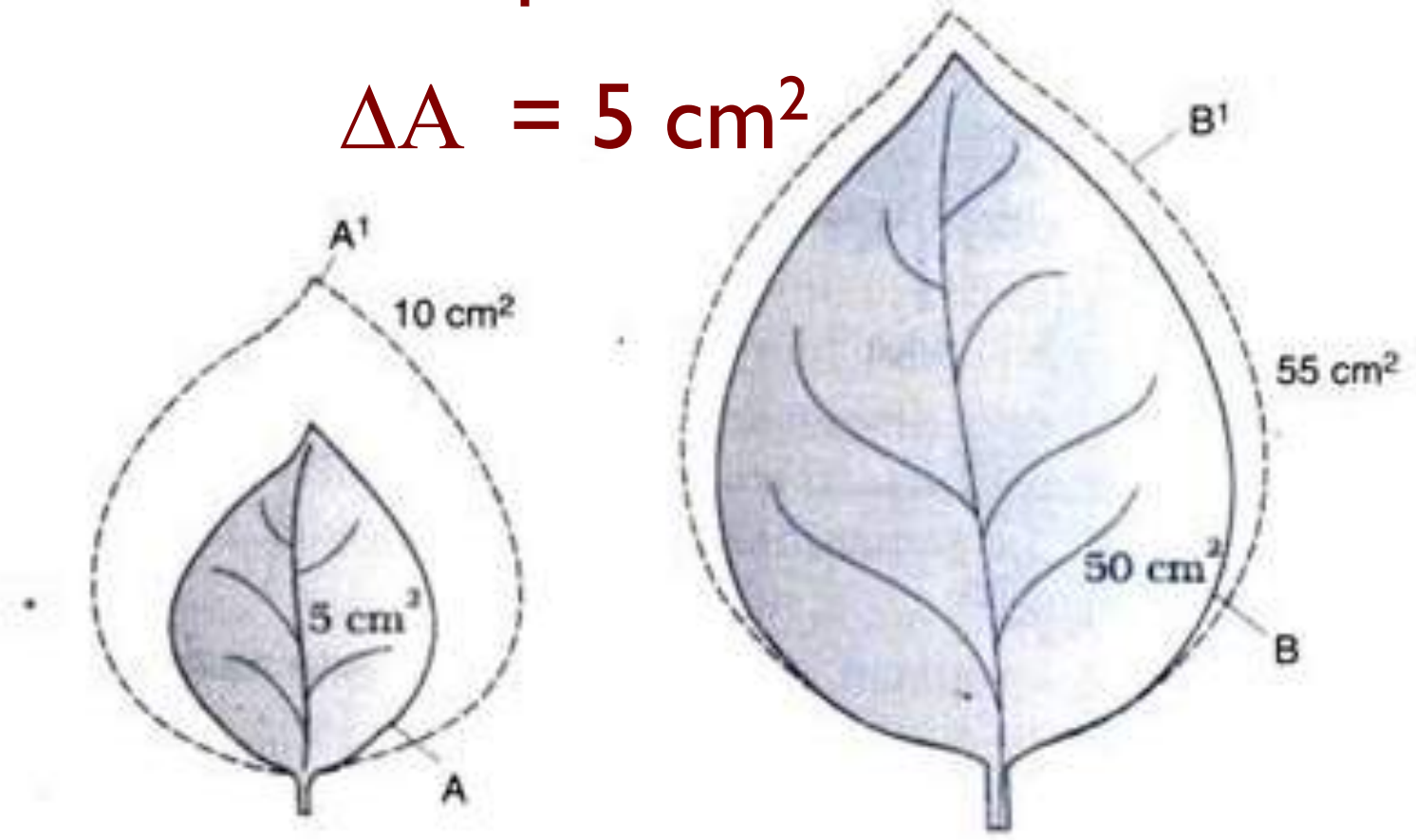
Differenze: una misura del cambiamento (cubi)

$12(a\text{-}2)$ $6(a\text{-}2)^2$ $(a\text{-}2)^3$

a		a		a		a		Δ1B	Δ1D	Δ2D	Δ1F	Δ2F	Δ3F
A	B	C	D	E	F	G	H			K	L	M	N
2	0	2	0	2	0	2	8	12	6	12	1	6	6
3	12	3	6	3	1	3	8	12	18	12	7	12	6
4	24	4	24	4	8	4	8	12	30	12	19	18	6
5	36	5	54	5	27	5	8	12	42	12	37	24	6
6	48	6	96	6	64	6	8	12	54	12	61	30	6
7	60	7	150	7	125	7	8	12	66	12	91	36	6
8	72	8	216	8	216	8	8	12	78	12	127	42	6
9	84	9	294	9	343	9	8	12	90	12	169	48	6
10	96	10	384	10	512	10	8	12	102	12	217	54	6
11	108	11	486	11	729	11	8	12	114	12	271	60	6
12	120	12	600	12	1000	12	8	12	126	12	331	66	6
13	132	13	726	13	1331	13	8	12	138	12	397	72	6
14	144	14	864	14	1728	14	8	12	150	12	469	78	6
15	156	15	1014	15	2197	15	8	12	162	12	547	84	6
16	168	16	1176	16	2744	16	8	12	174	12	631	90	6
17	180	17	1350	17	3375	17	8	12	186	12	721	96	6
18	192	18	1536	18	4096	18	8	12	198	12	817	102	6
19	204	19	1734	19	4913	19	8	12	210	12	919	108	6
20	216	20	1944	20	5832	20	8	12	222	12	1027	114	6
21	228	21	2166	21	6859	21	8	12	234	12	1141	120	6
22	240	22	2400	22	8000	22	8	12	246	12	1261	126	6
23	252	23	2646	23	9261	23	8	12	258	12	1387	132	6
24	264	24	2904	24	10648	24	8	12	270	12	1519	138	6
25	276	25	3174	25	12167	25	8	12	282	12	1657	144	6
26	288	26	3456	26	13824	26	8	12	294	12	1801	150	
27	300	27	3750	27	15625	27	8	12	306		1951		

Un'idea più fine del cambiamento

$\Delta A = 5\ cm^2$



Il cambiamento relativo $\Delta_r A = \Delta A/A$

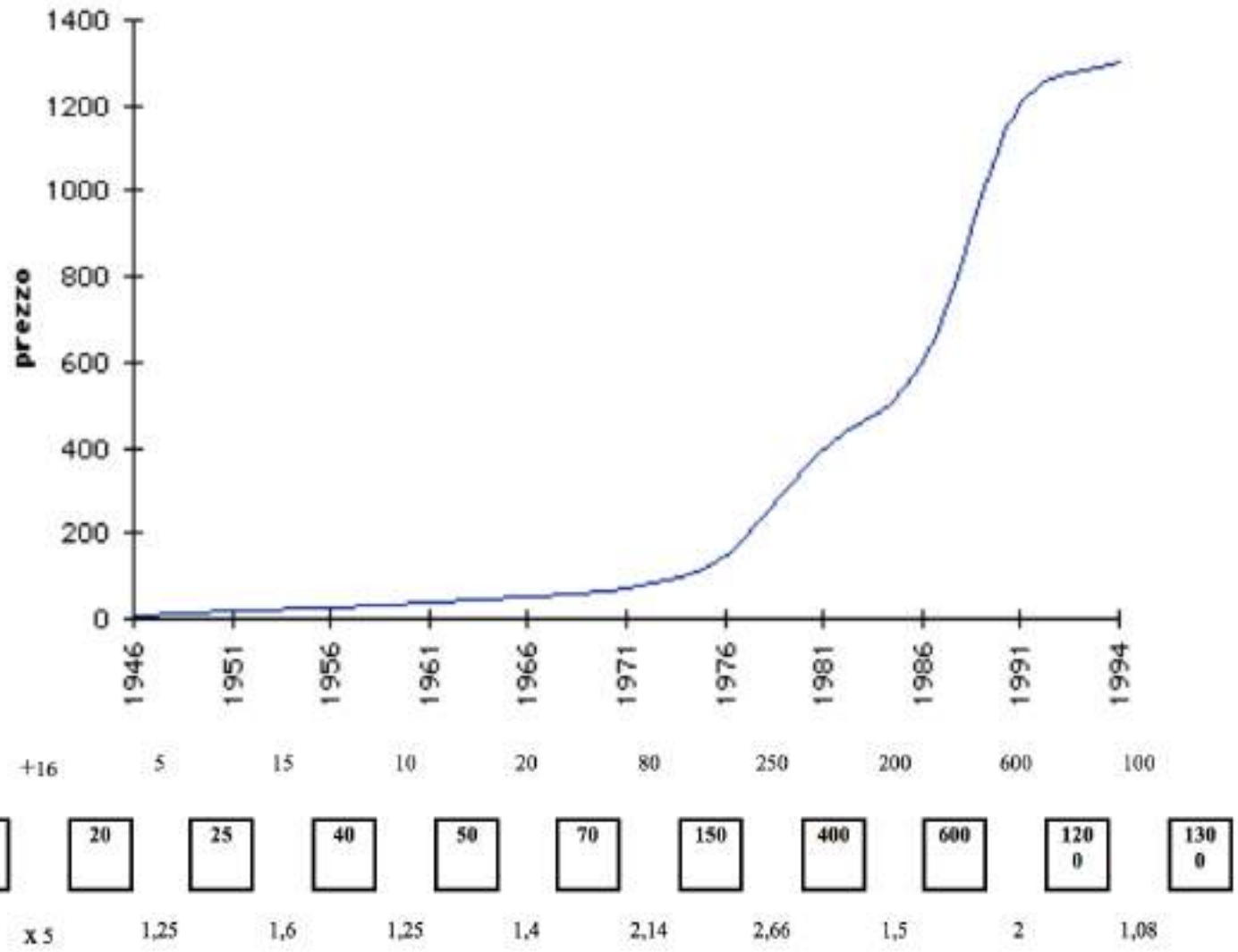
$\Delta_r = 5\ cm^2 / 5\ cm^2$

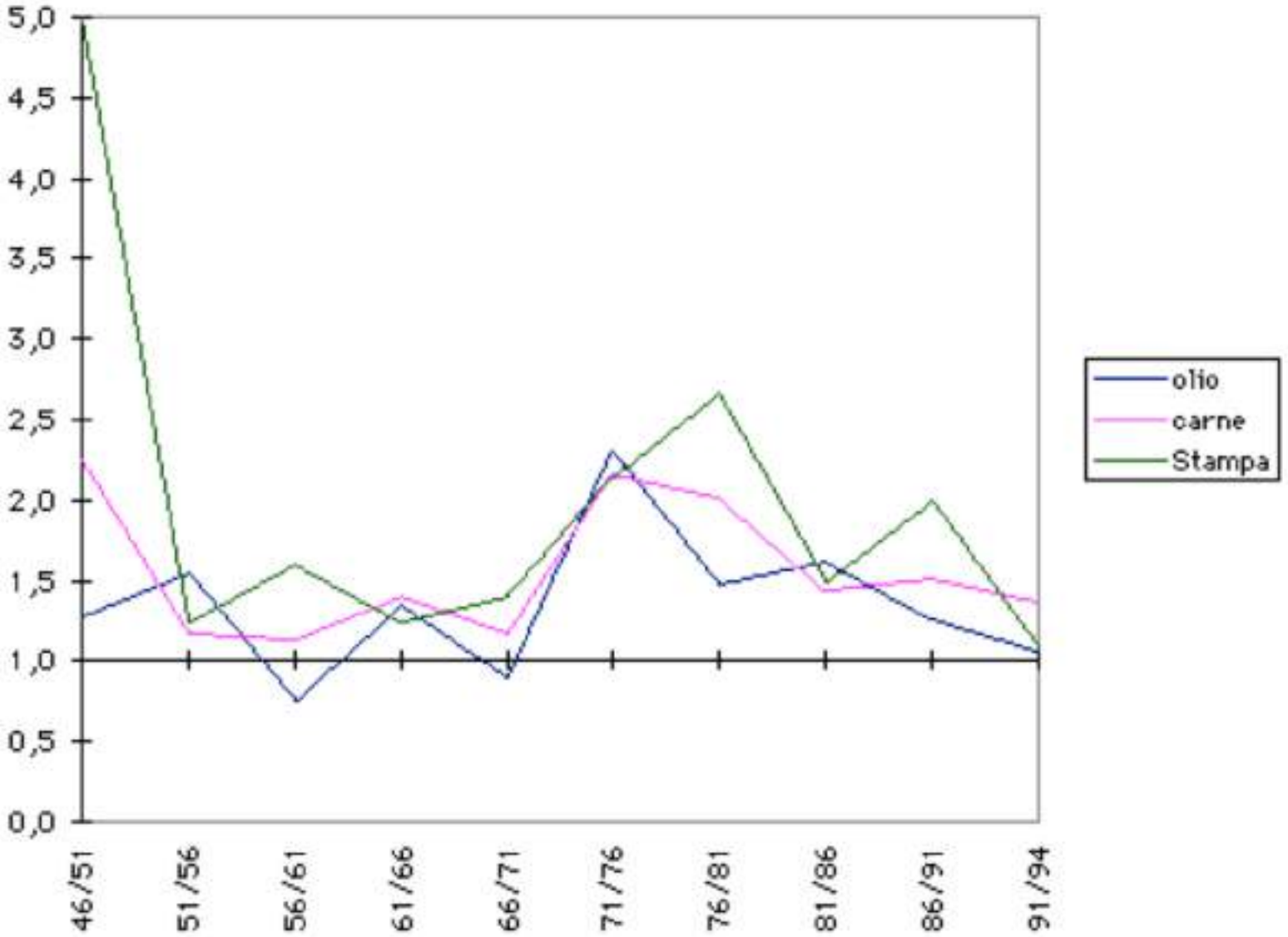
100%

$\Delta_r = 5\ cm^2 / 50\ cm^2$

10%

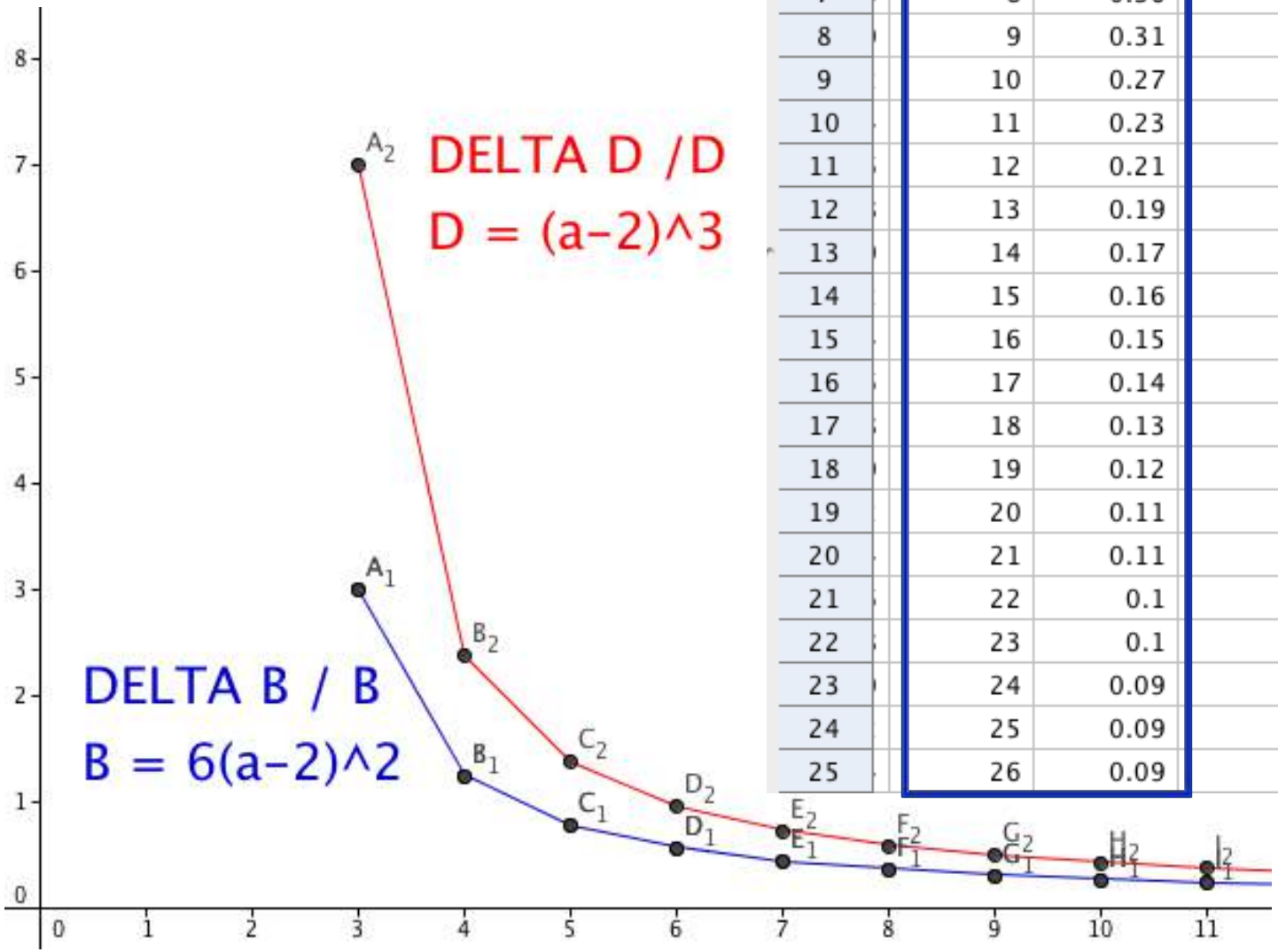
Esempio 8. Il valore del denaro nel tempo



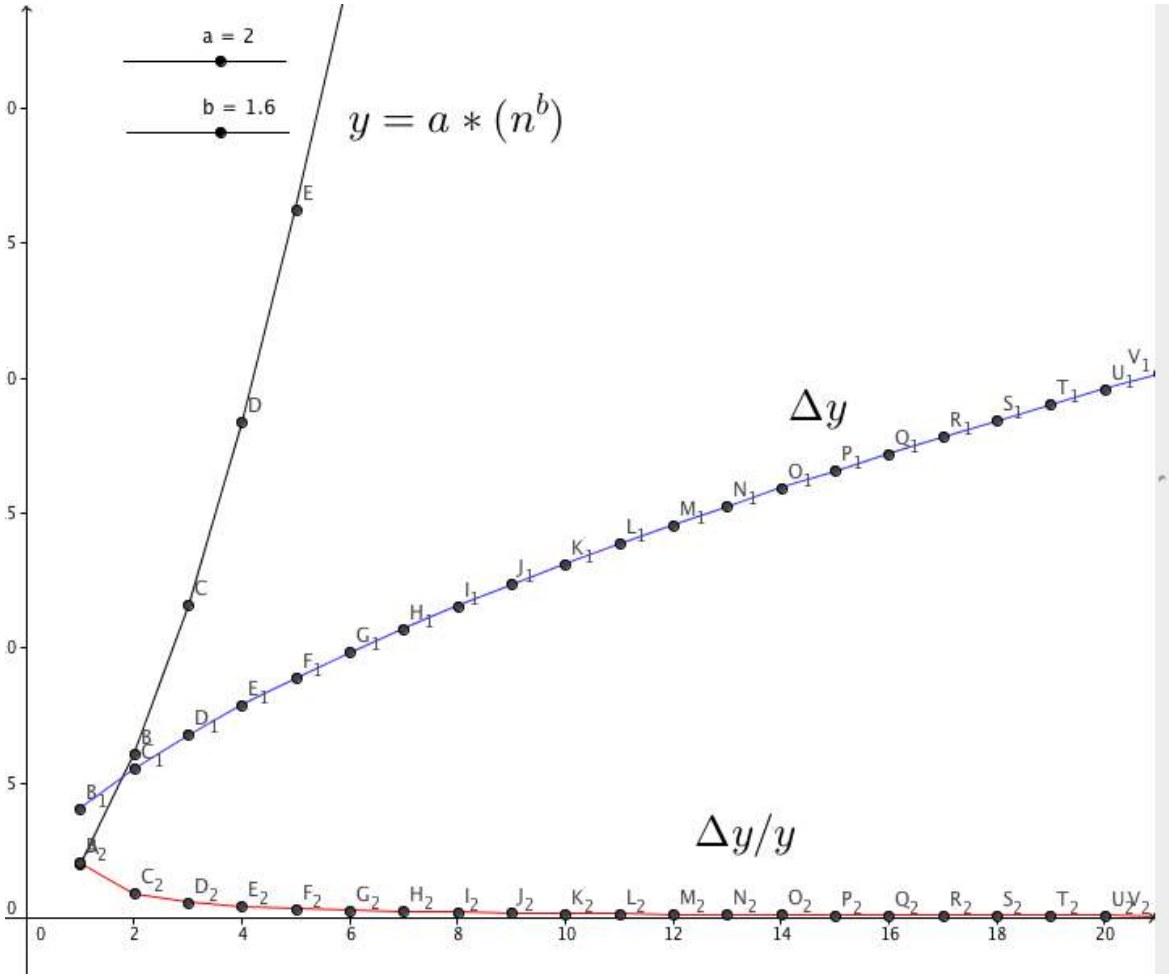

5,0
4,5
4,0
3,5
3,0
2,5
2,0
1,5
1,0
0,5
0,0
46/51
51/56
56/61
61/66
66/71
71/76
76/81
81/86
86/91
91/94
olio
carne
Stampa

Cubi e differenze relative

		ΔB/B				ΔD/D	
1	2		2	1	2		
2	3	3	3	7	3	7	
3	4	1.25	4	19	4	2.38	
4	5	0.78	5	37	5	1.37	
5	6	0.56	6	61	6	0.95	
6	7	0.44	7	91	7	0.73	
7	8	0.36	8	127	8	0.59	
8	9	0.31	9	169	9	0.49	
9	10	0.27	10	217	10	0.42	
10	11	0.23	11	271	11	0.37	
11	12	0.21	12	331	12	0.33	
12	13	0.19	13	397	13	0.3	
13	14	0.17	14	469	14	0.27	
14	15	0.16	15	547	15	0.25	
15	16	0.15	16	631	16	0.23	
16	17	0.14	17	721	17	0.21	
17	18	0.13	18	817	18	0.2	
18	19	0.12	19	919	19	0.19	
19	20	0.11	20	1027	20	0.18	
20	21	0.11	21	1141	21	0.17	
21	22	0.1	22	1261	22	0.16	
22	23	0.1	23	1387	23	0.15	
23	24	0.09	24	1519	24	0.14	
24	25	0.09	25	1657	25	0.14	
25	26	0.09	26	1801			

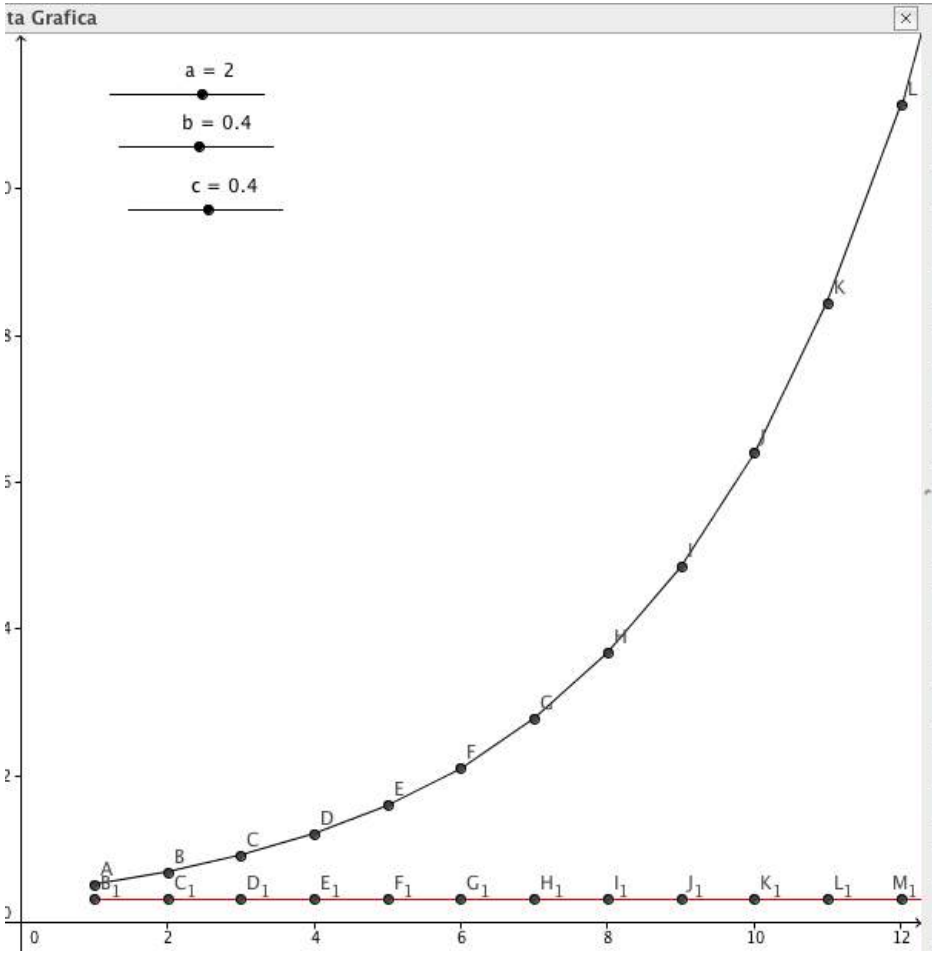


Diff relative: polinomi



	A	B	C	D	E	F
1	n	a*n^b	n	DELTA B	n	DELTA B /B
2	1	2	1	4.06	1	2.03
3	2	6.06	2	5.54	2	0.91
4	3	11.6	3	6.78	3	0.58
5	4	18.38	4	7.89	4	0.43
6	5	26.27	5	8.9	5	0.34
7	6	35.16	6	9.84	6	0.28
8	7	45	7	10.72	7	0.24
9	8	55.72	8	11.55	8	0.21
10	9	67.27	9	12.35	9	0.18
11	10	79.62	10	13.12	10	0.16
12	11	92.74	11	13.85	11	0.15
13	12	106.59	12	14.56	12	0.14
14	13	121.15	13	15.25	13	0.13
15	14	136.41	14	15.92	14	0.12
16	15	152.33	15	16.57	15	0.11
17	16	168.9	16	17.2	16	0.1
18	17	186.1	17	17.82	17	0.1
19	18	203.92	18	18.43	18	0.09
20	19	222.35	19	19.02	19	0.09
21	20	241.37	20	19.6	20	0.08
22	21	260.96	21	20.17	21	0.08
23	22	281.13	22	20.72	22	0.07
24	23	301.85	23	21.27	23	0.07
25	24	323.12	24	21.81	24	0.07
26	25	344.93	25		25	

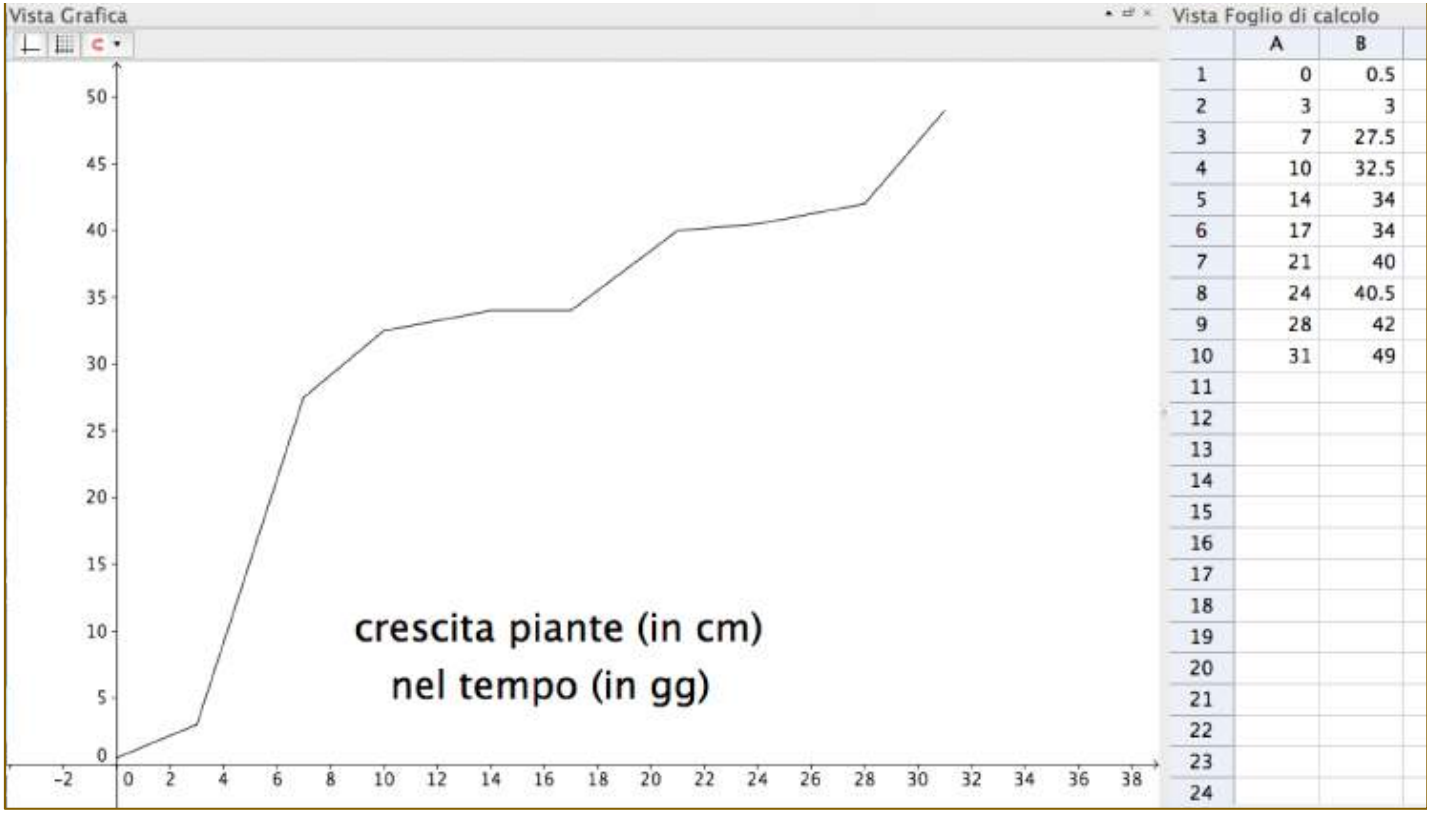
Diff relative: esponenziali



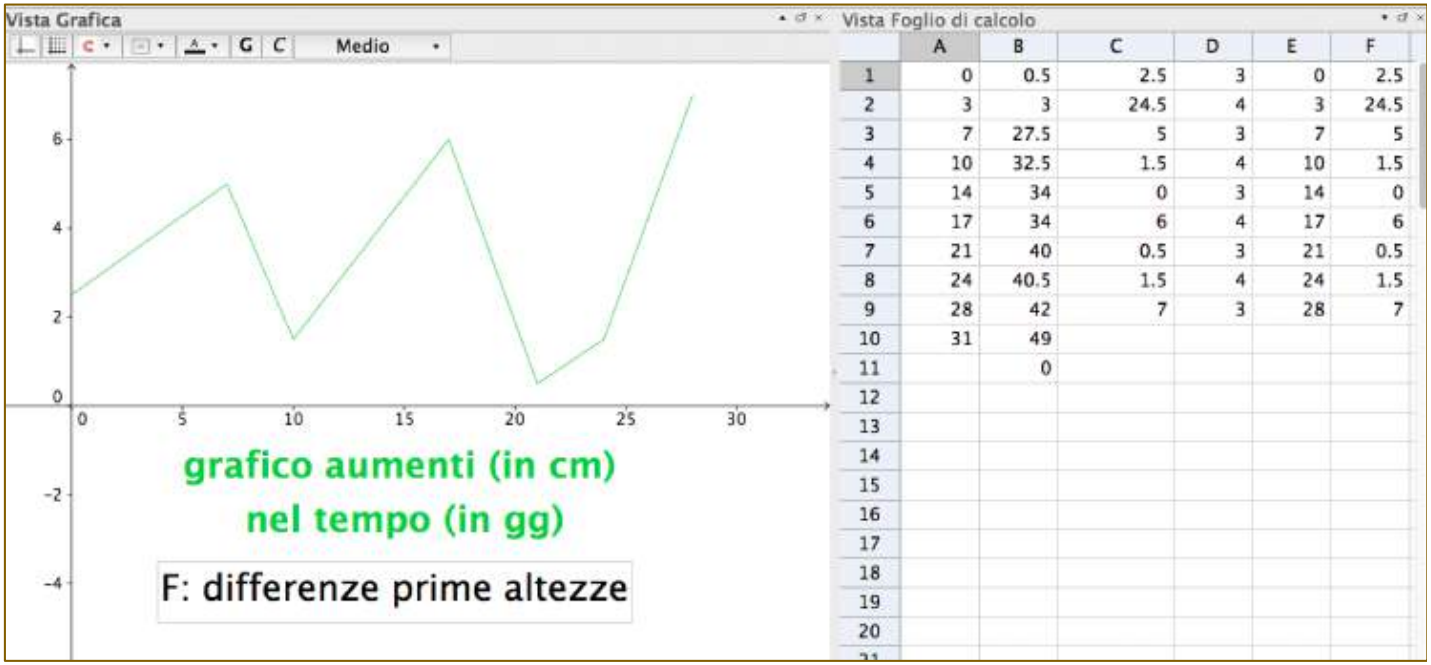
Vista Foglio di calcolo

	A	B	C	D	E	F	G	H
1	n	a^n	a^(bn)	n	c*a^(b*n)	DELTA E	n	DELTA E/E
2	1	2	1.32	1	0.53	0.17	1	0.32
3	2	4	1.74	2	0.7	0.22	2	0.32
4	3	8	2.3	3	0.92	0.29	3	0.32
5	4	16	3.03	4	1.21	0.39	4	0.32
6	5	32	4	5	1.6	0.51	5	0.32
7	6	64	5.28	6	2.11	0.67	6	0.32
8	7	128	6.96	7	2.79	0.89	7	0.32
9	8	256	9.19	8	3.68	1.17	8	0.32
10	9	512	12.13	9	4.85	1.55	9	0.32
11	10	1024	16	10	6.4	2.04	10	0.32
12	11	2048	21.11	11	8.44	2.7	11	0.32
13	12	4096	27.86	12	11.14	3.56	12	0.32
14	13	8192	36.76	13	14.7	4.7	13	0.32
15	14	16384	48.5	14	19.4	6.2	14	0.32
16	15	32768	64	15	25.6	8.18	15	0.32
17	16	65536	84.45	16	33.78	10.79	16	0.32
18	17	131072	111.43	17	44.57	14.24	17	0.32
19	18	262144	147.03	18	58.81	18.79	18	0.32
20	19	524288	194.01	19	77.6	24.8	19	0.32
21	20	1048576	256	20	102.4	32.72	20	0.32
22	21	2097152	337.79	21	135.12	43.17	21	0.32
23	22	4194304	445.72	22	178.29	56.96	22	0.32
24	23	8388608	588.13	23	235.25	75.17	23	0.32
25	24	167772...	776.05	24	310.42	99.18	24	0.32
26	25	335544...	1024	25	409.6			

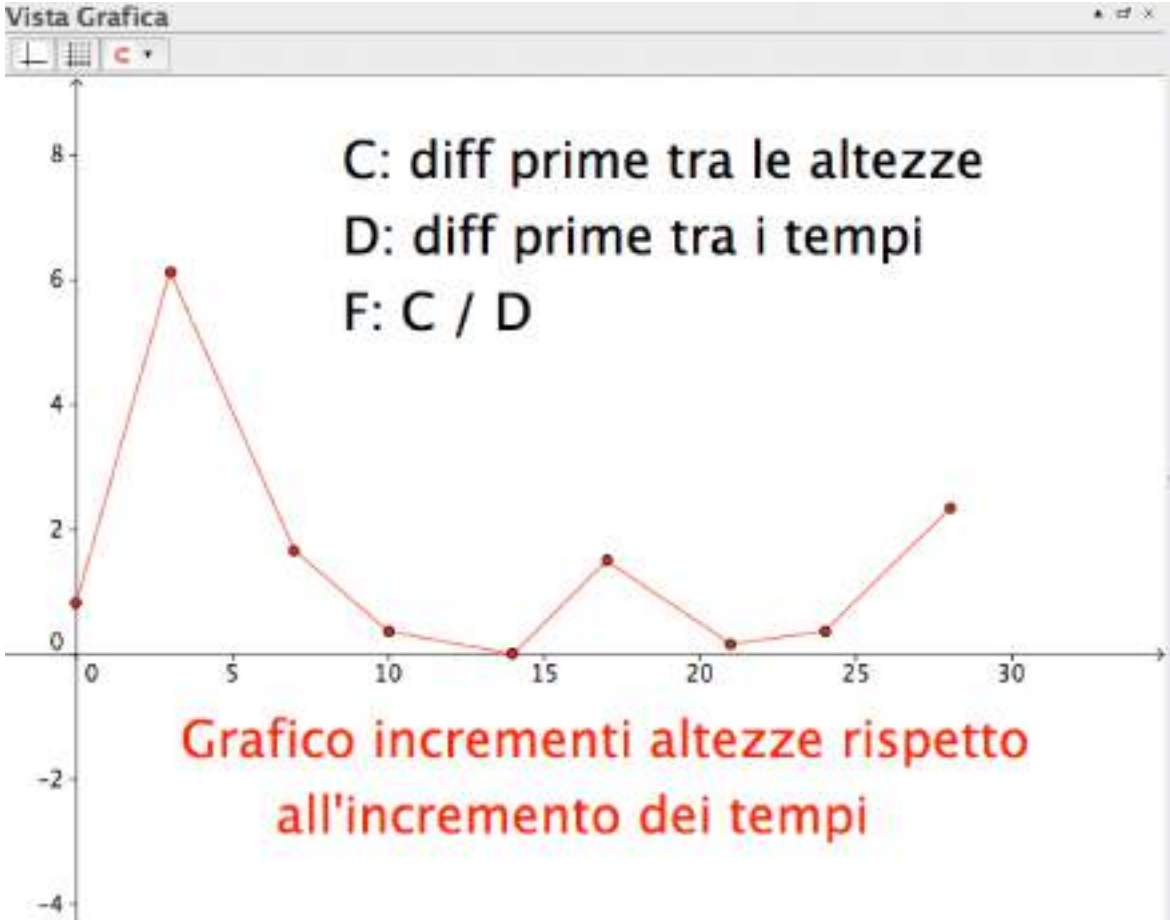
Crescita fagioli



	A	B
1	0	0.5
2	3	3
3	7	27.5
4	10	32.5
5	14	34
6	17	34
7	21	40
8	24	40.5
9	28	42
10	31	49
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		



	A	B	C	D	E	F
1	0	0.5	2.5	3	0	2.5
2	3	3	24.5	4	3	24.5
3	7	27.5	5	3	7	5
4	10	32.5	1.5	4	10	1.5
5	14	34	0	3	14	0
6	17	34	6	4	17	6
7	21	40	0.5	3	21	0.5
8	24	40.5	1.5	4	24	1.5
9	28	42	7	3	28	7
10	31	49				
11		0				
12						
13						
14						
15						
16						
17						
18						
19						
20						

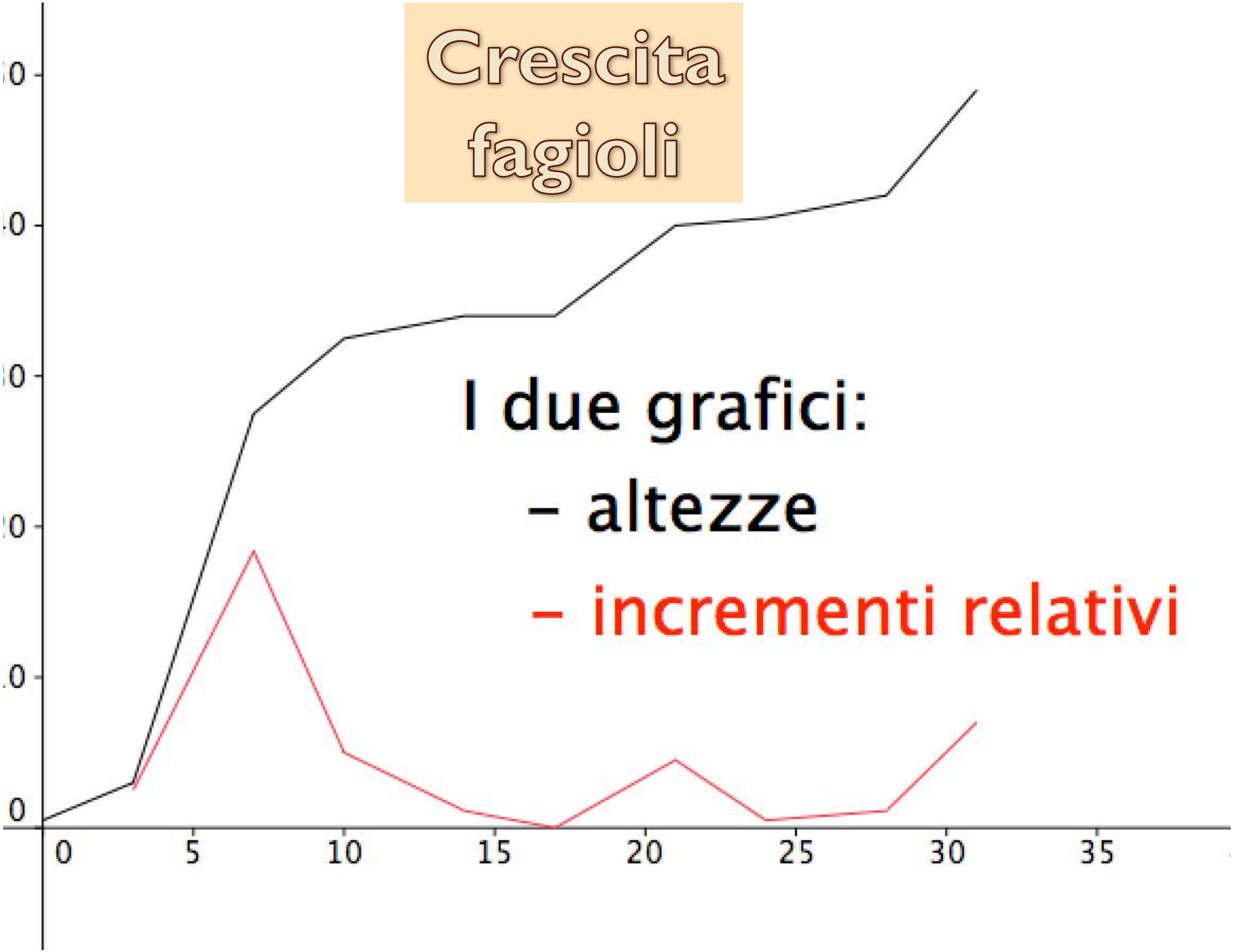


Vista Foglio di calcolo

	A	B	C	D	E	F
1	0	0.5	2.5	3	0	0.83
2	3	3	24.5	4	3	6.13
3	7	27.5	5	3	7	1.67
4	10	32.5	1.5	4	10	0.38
5	14	34	0	3	14	0
6	17	34	6	4	17	1.5
7	21	40	0.5	3	21	0.17
8	24	40.5	1.5	4	24	0.38
9	28	42	7	3	28	2.33
10	31	49				
11		0				
12						
13						
14						
15						
16						
17						
18						
19						
20						

Crescita fagioli

Crescita fagioli



Crescita fagioli

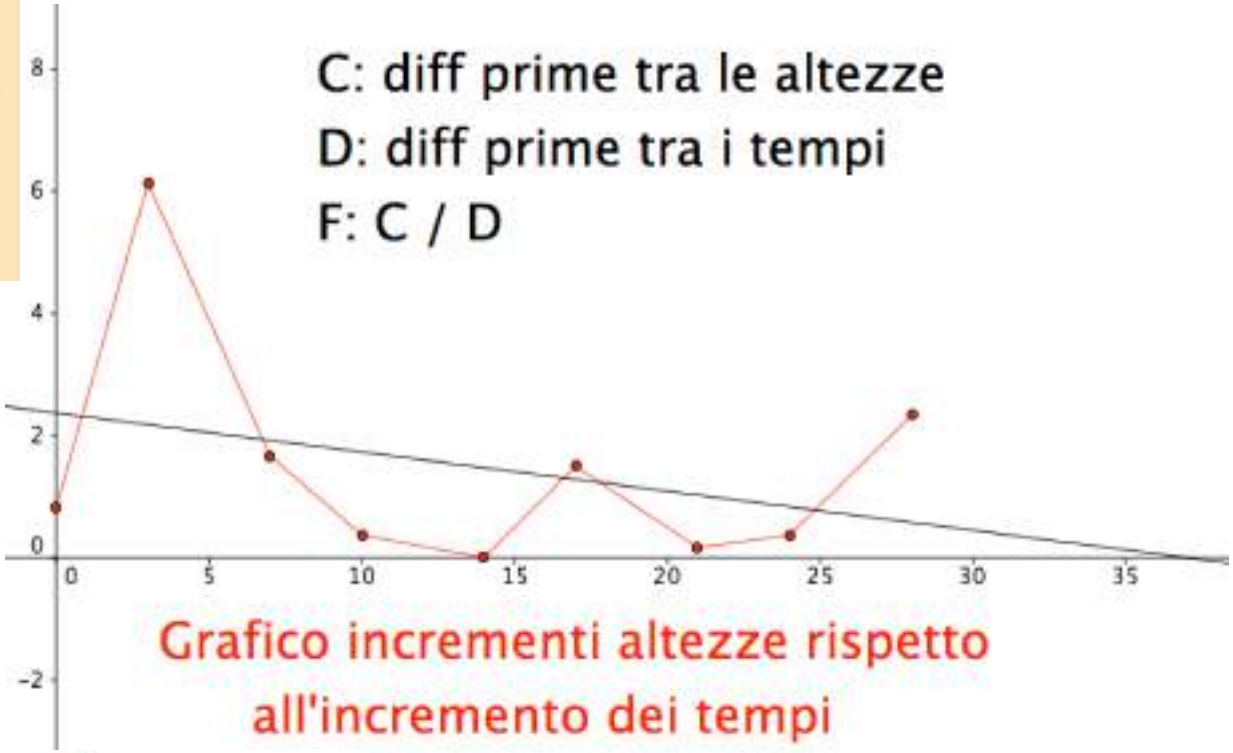
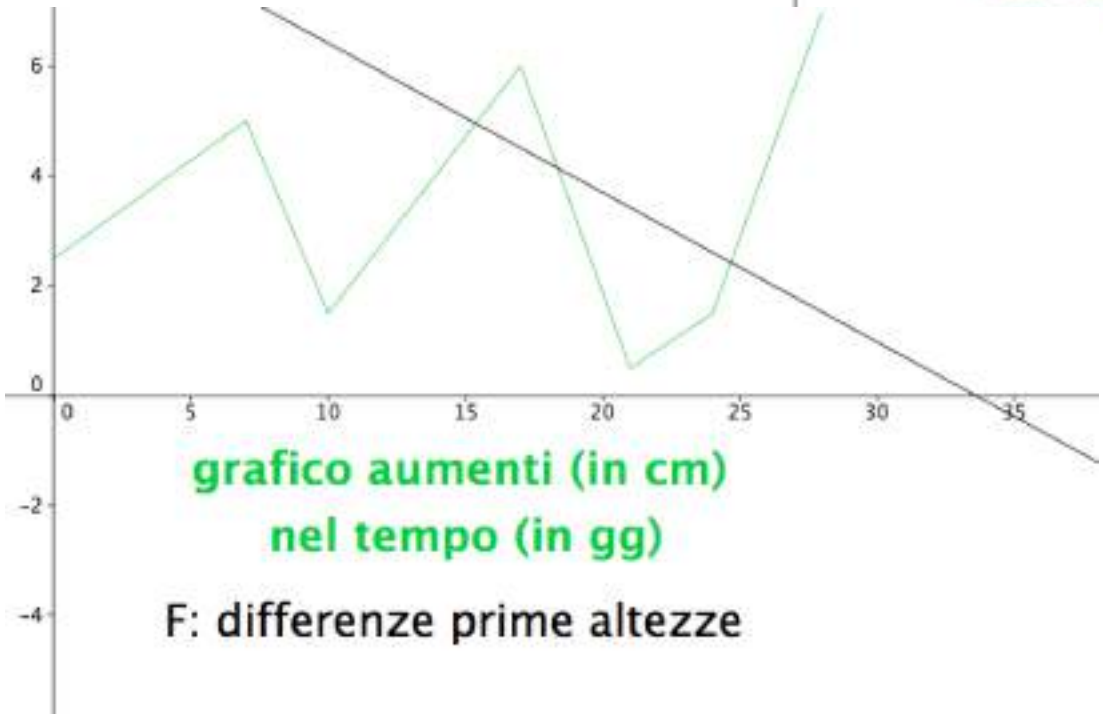
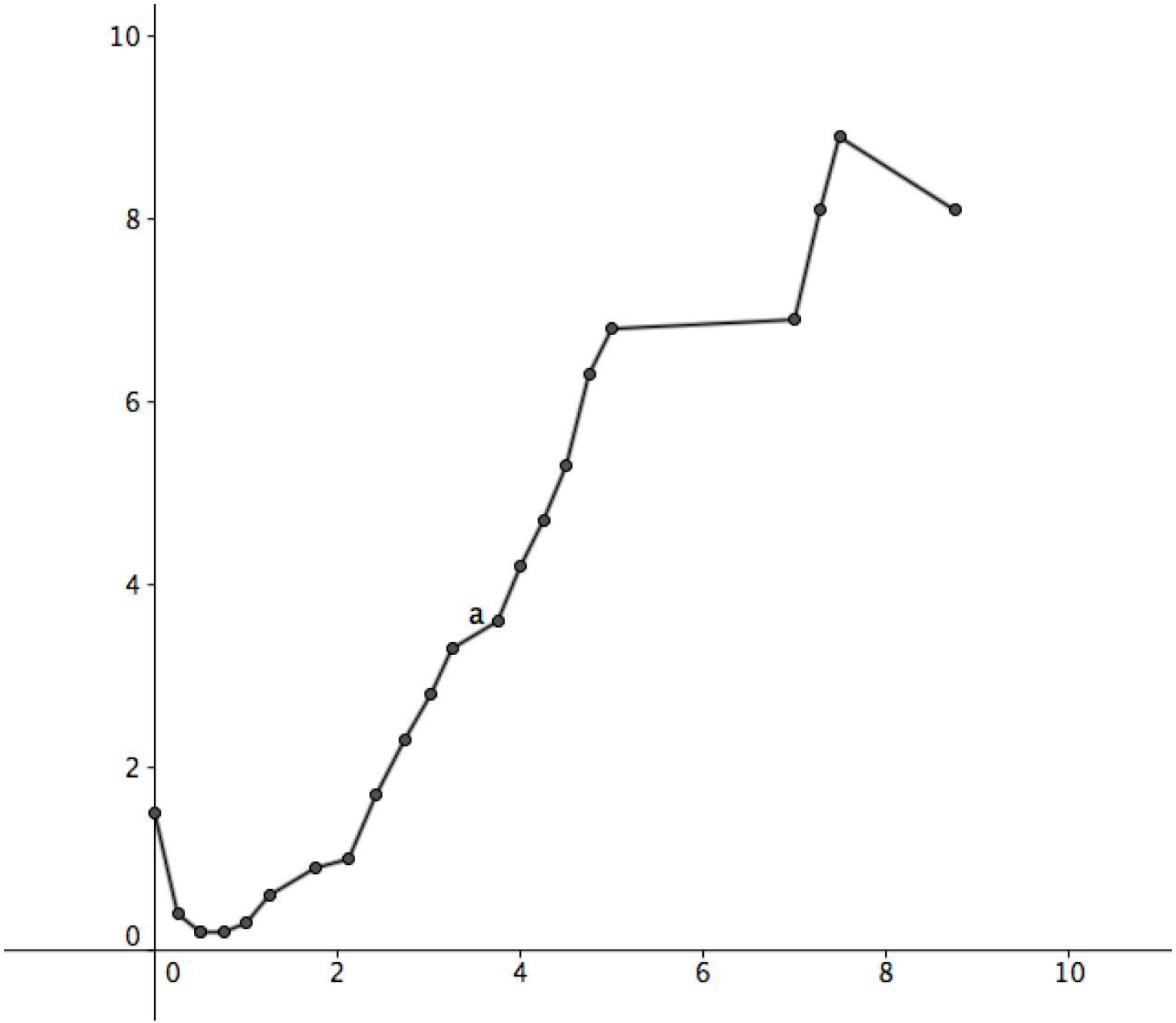


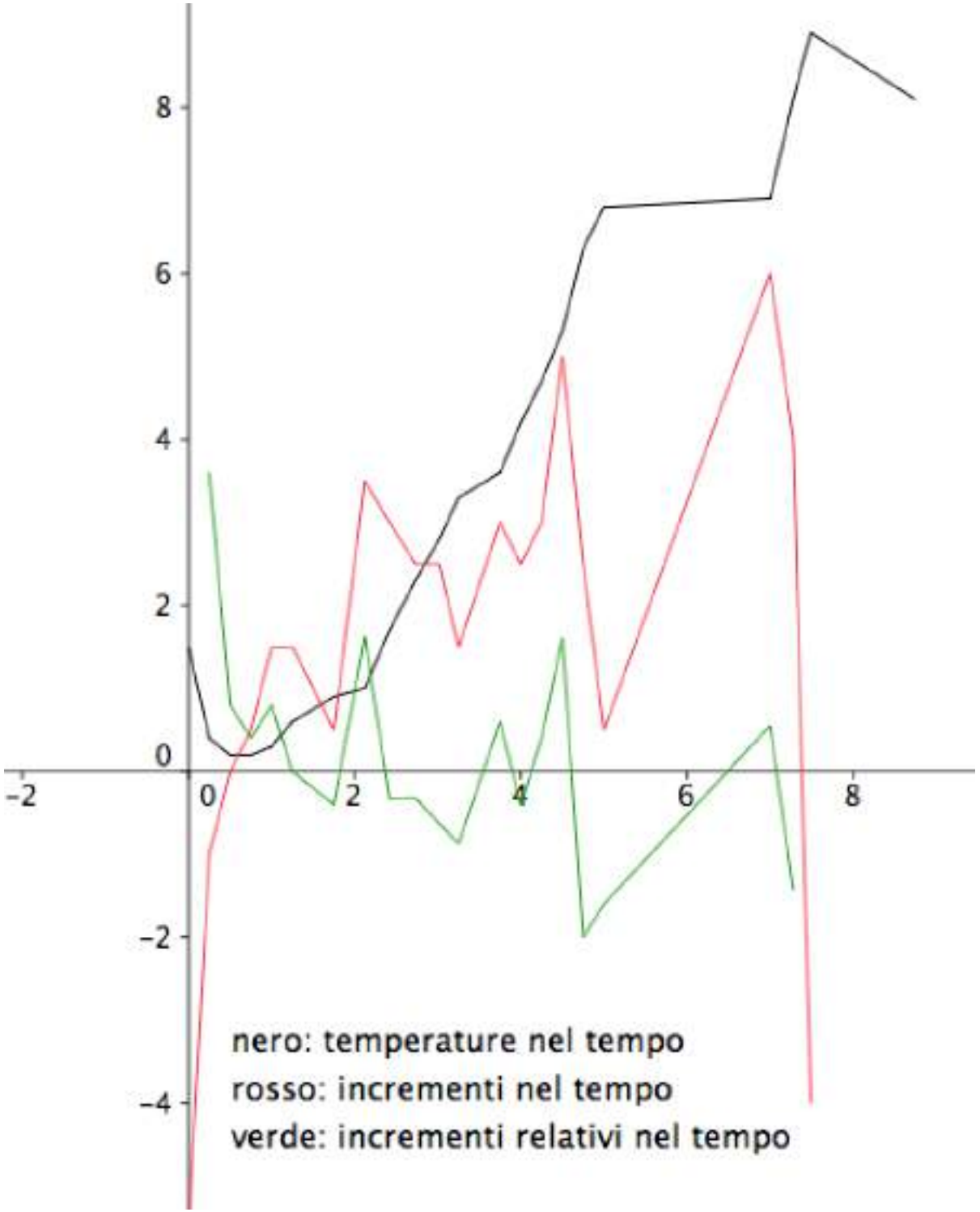
Grafico incrementi altezze rispetto all'incremento dei tempi



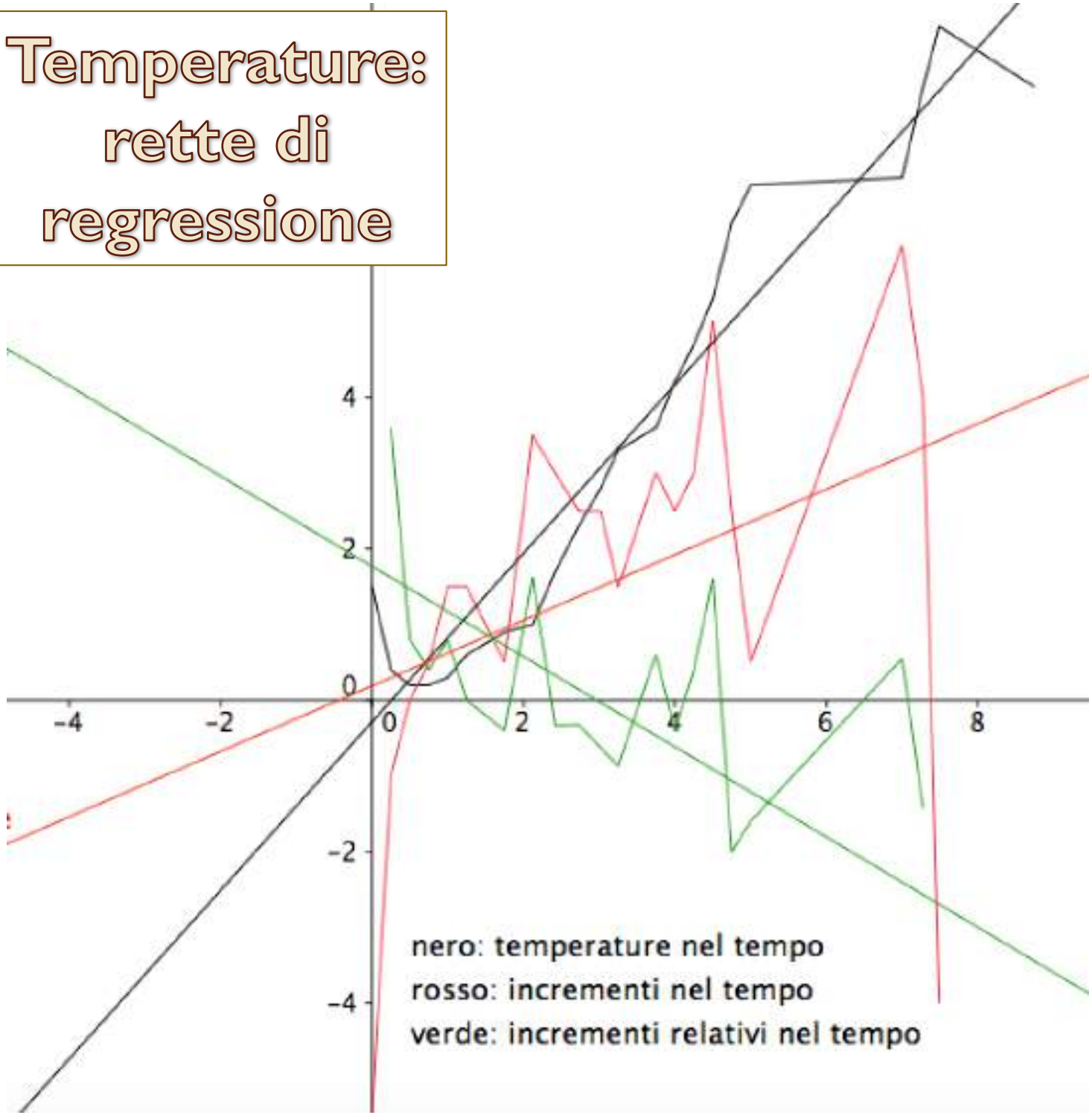
Rette di regressione

Temperature



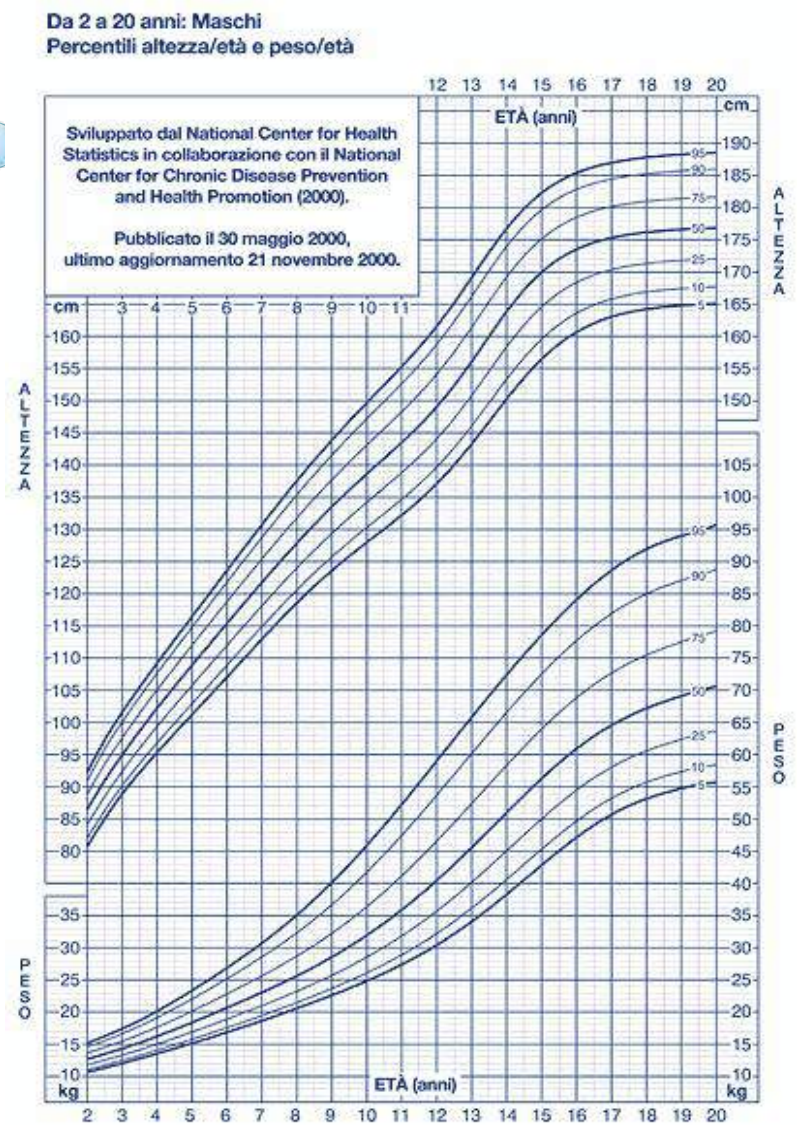

8
6
4
2
0
-2
-4
-2
0
2
4
6
8
nero: temperature nel tempo
rosso: incrementi nel tempo
verde: incrementi relativi nel tempo

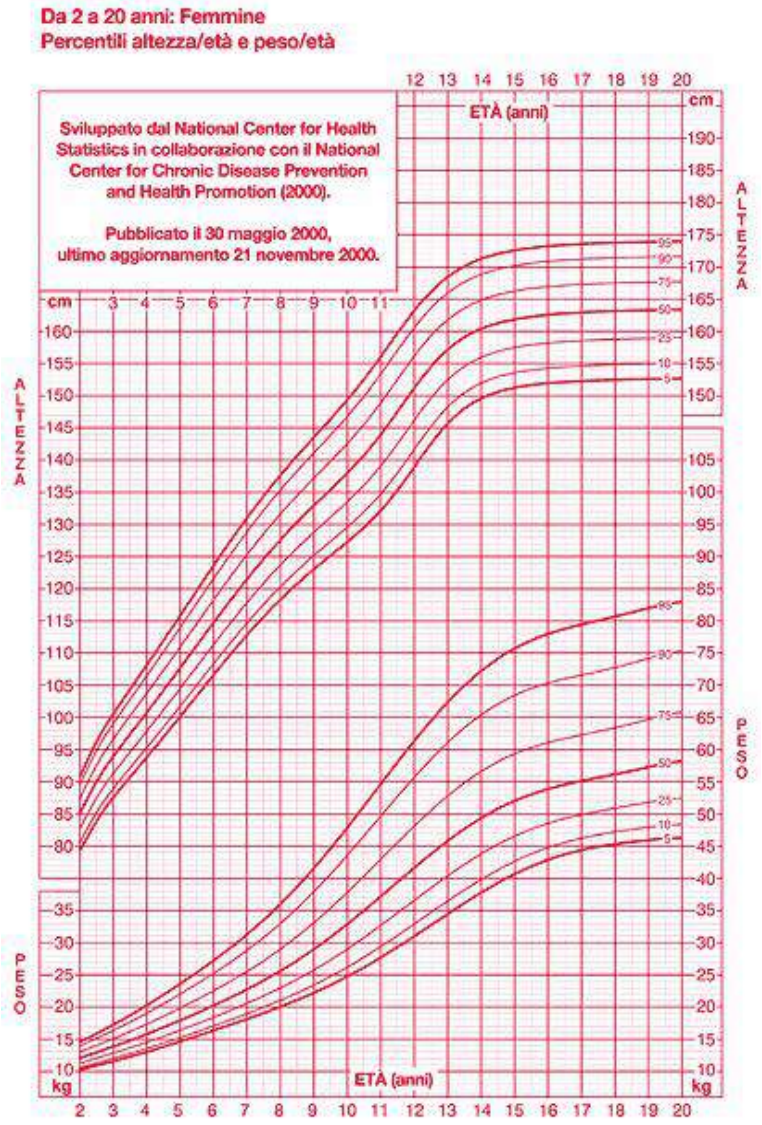
Temperature: rette di regressione



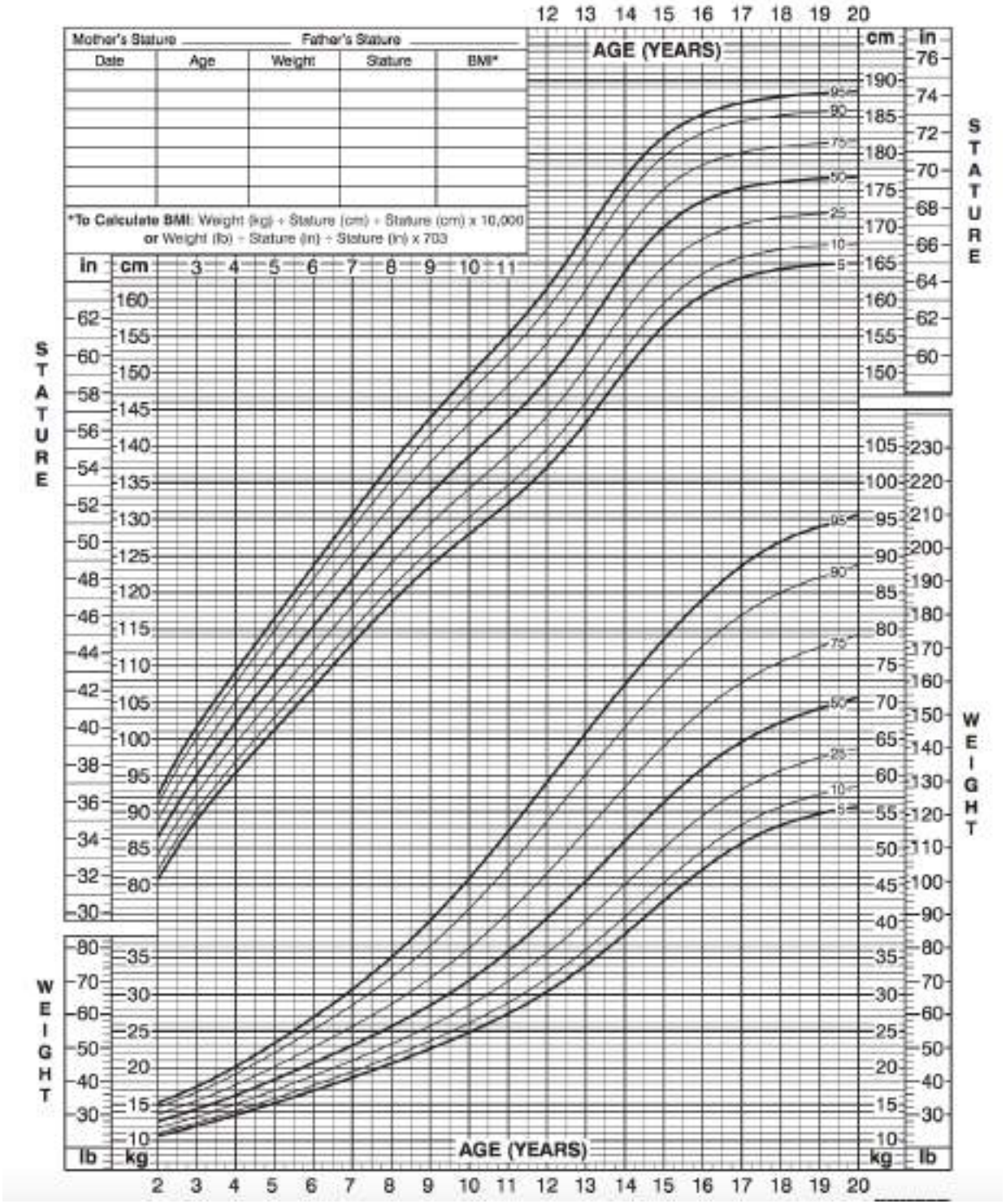
Altri fenomeni di crescita

Crescita media ragazze/i

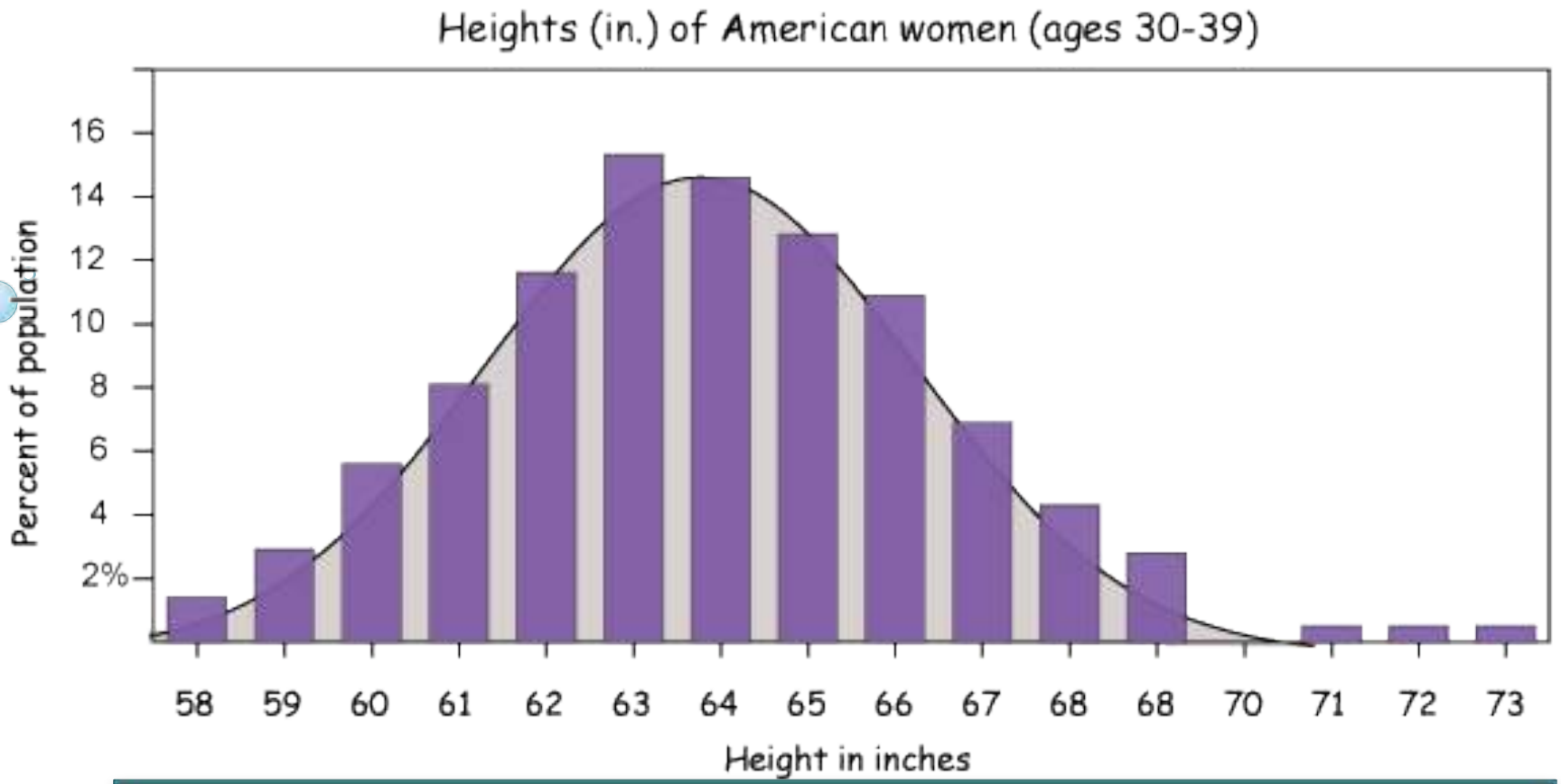




Ragazzi

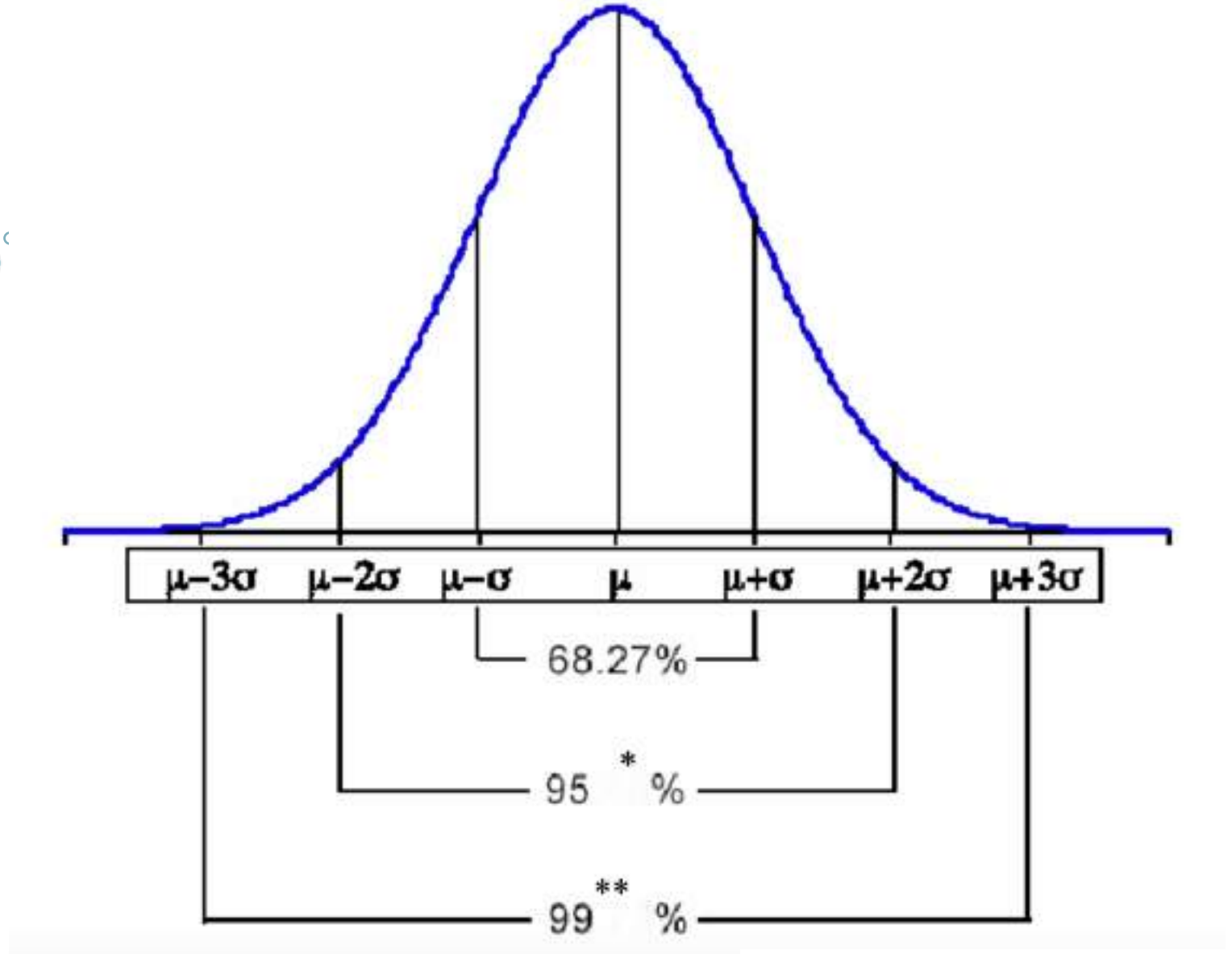

Mother's Stature
Father's Stature
Date
Age
Weight
Stature
BMI*
*To Calculate BMI: Weight (kg) ÷ Stature (cm) ÷ Stature (cm) x 10,000
or Weight (lb) ÷ Stature (in) ÷ Stature (in) x 703
AGE (YEARS)
STATURE
WEIGHT
95th
90
75
50
25
10
5
in
cm
lb
kg

CDC



147	152	157	160	167	173	185

Altezza in cm

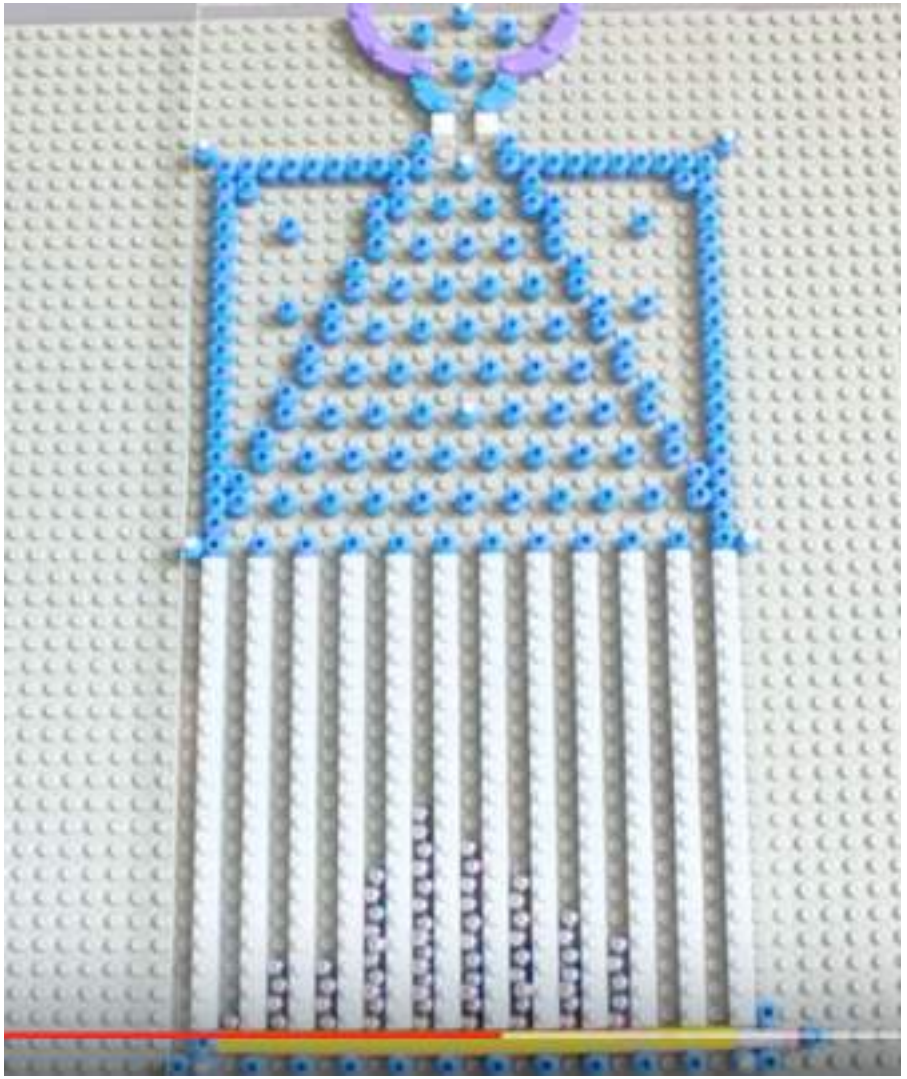


$$\sigma_X = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N}},$$

dove $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$ è la media aritmetica di X.

μ indica la media aritmetica dei valori

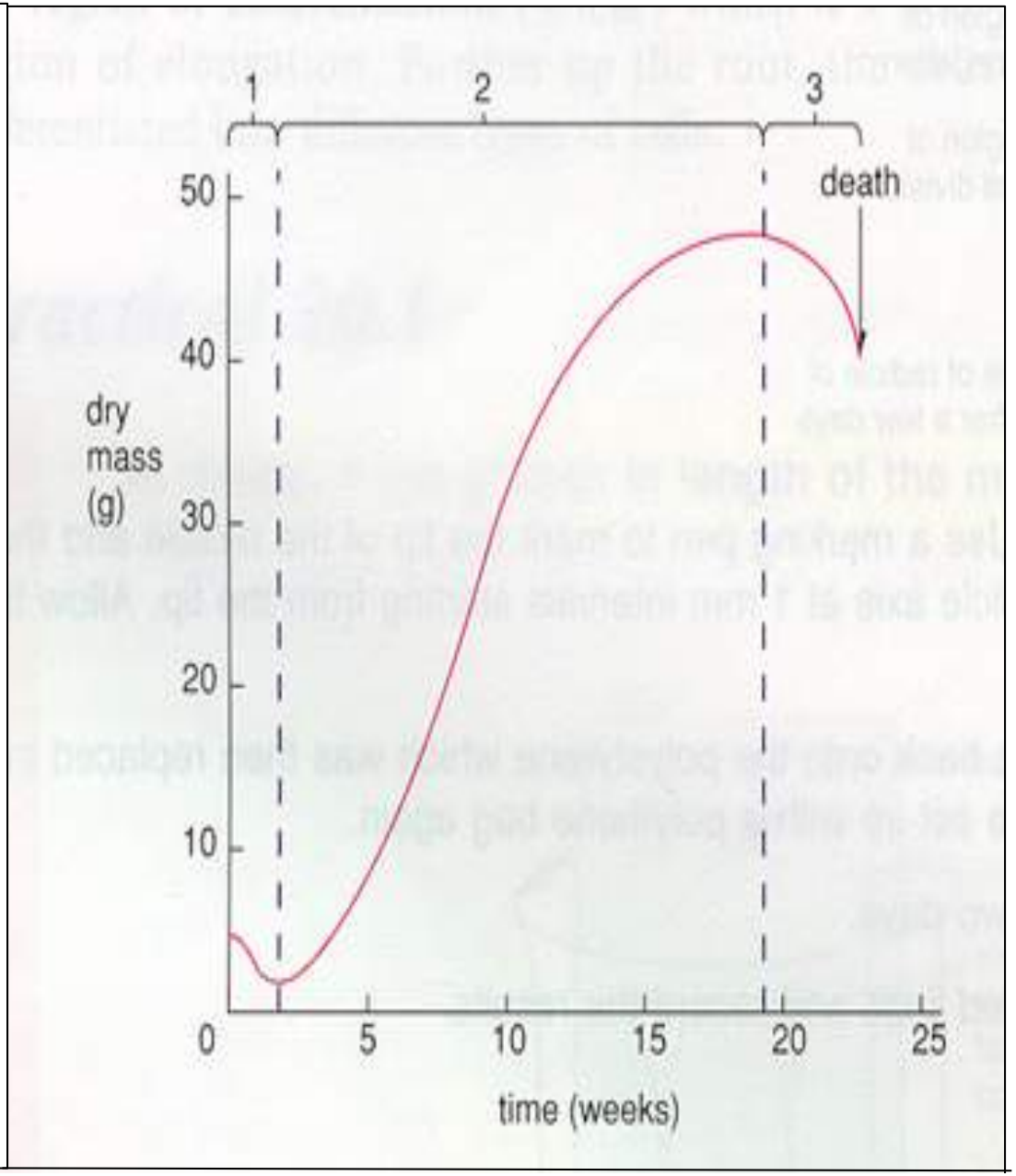
Macchina di Galton



La curva della crescita delle piante

Le tre fasi nella crescita delle piante annuali:

1. La massa secca diminuisce durante la fase iniziale della germinazione perché la pianta consuma le riserve alimentari del seme
2. La massa cresce decisamente perché le foglie producono l'alimentazione necessaria con la fotosintesi
3. La massa decresce per la produzione dei semi o dei frutti

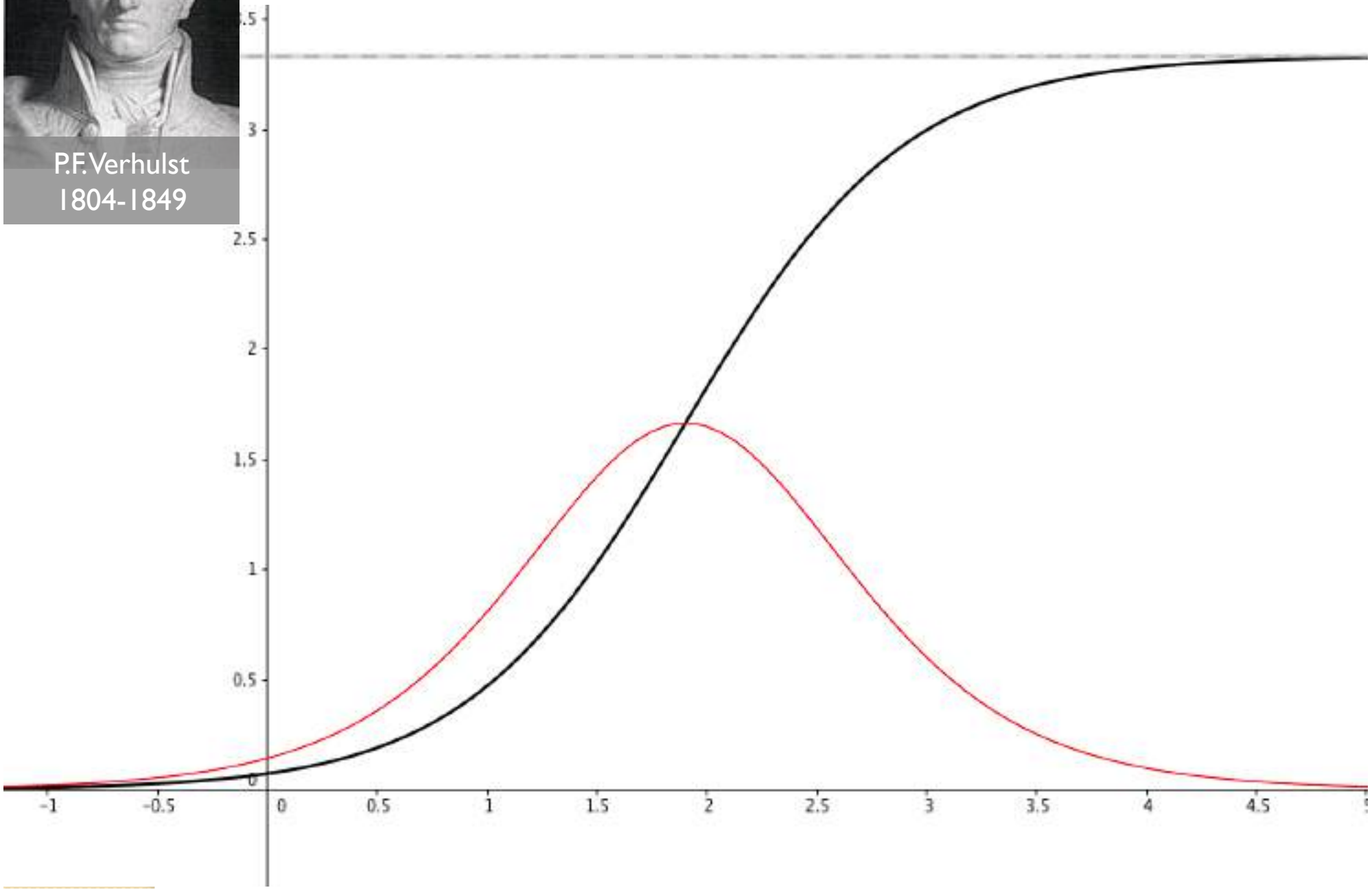


Curva logistica

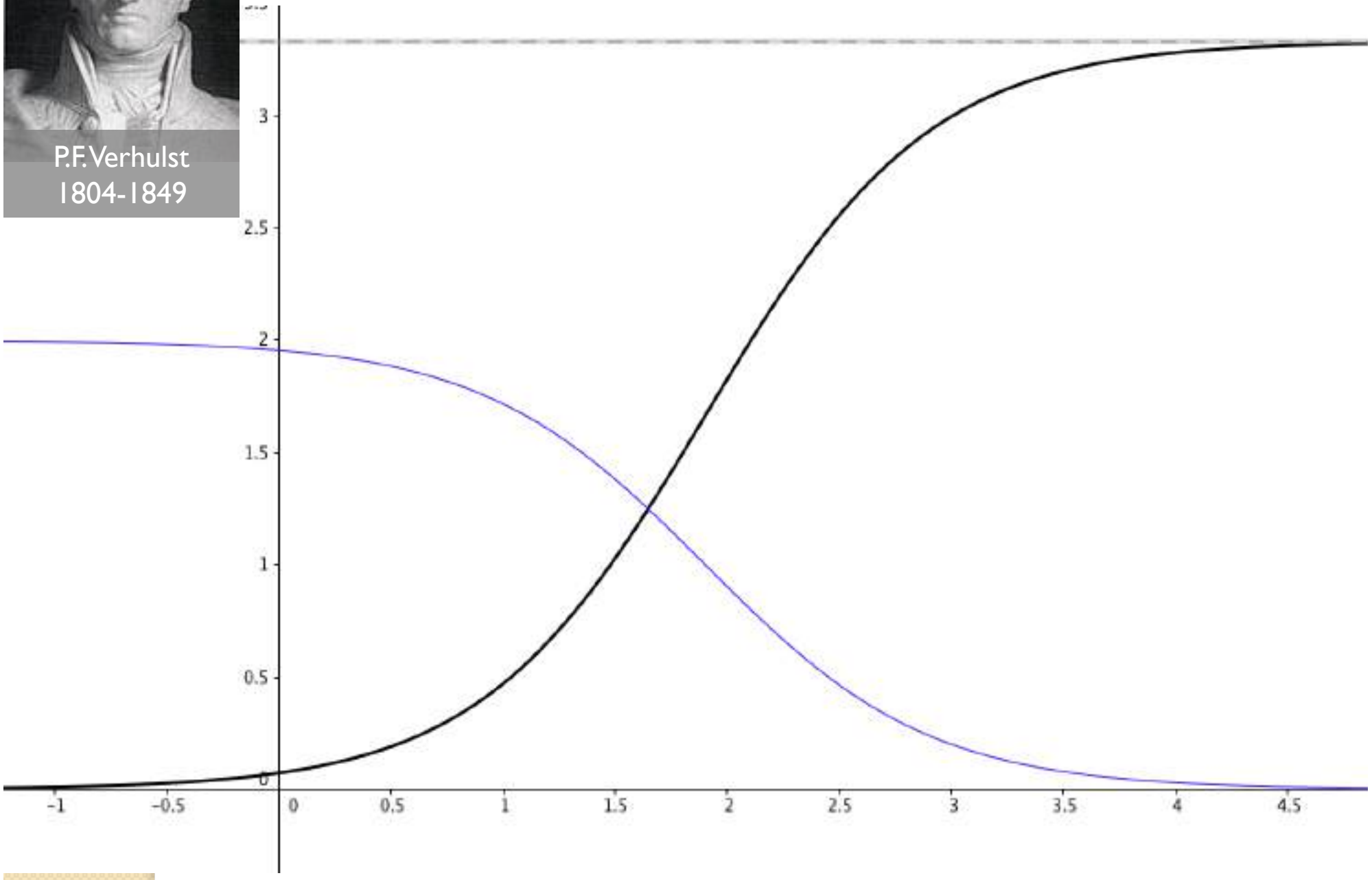
P.F. Verhulst
1804-1849

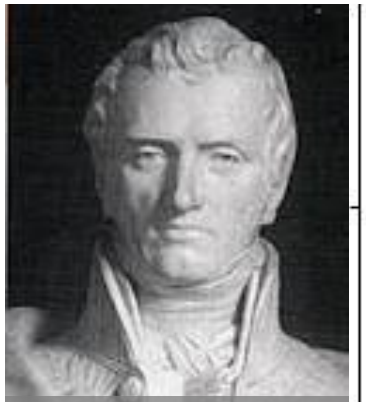
limite risorse

P.F. Verhulst
1804-1849



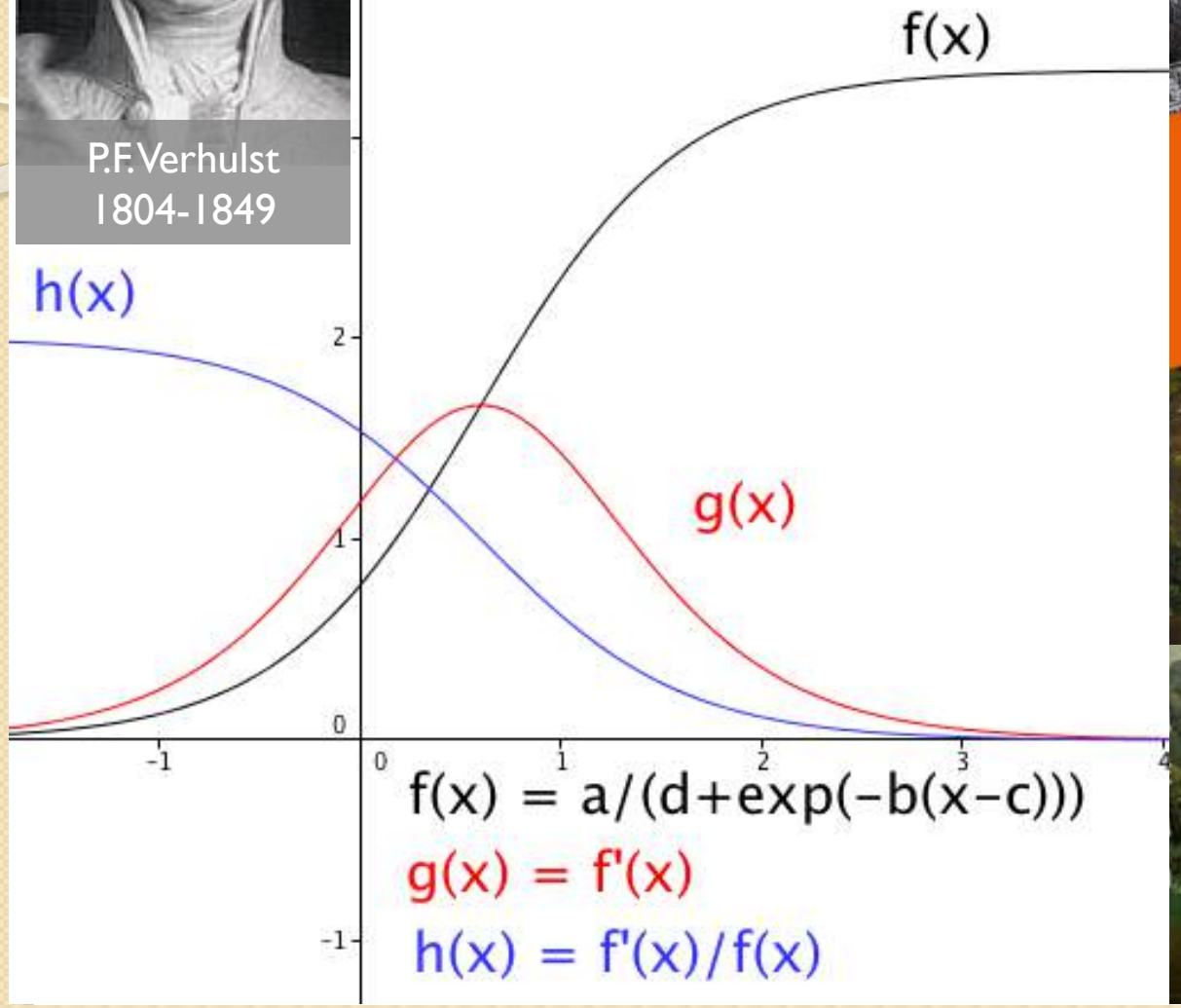
P.F. Verhulst
1804-1849

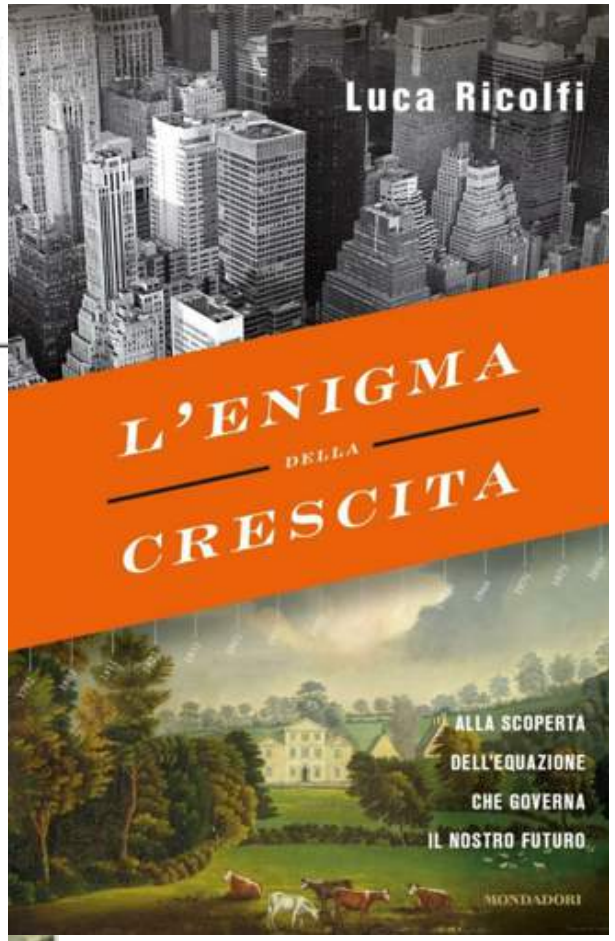




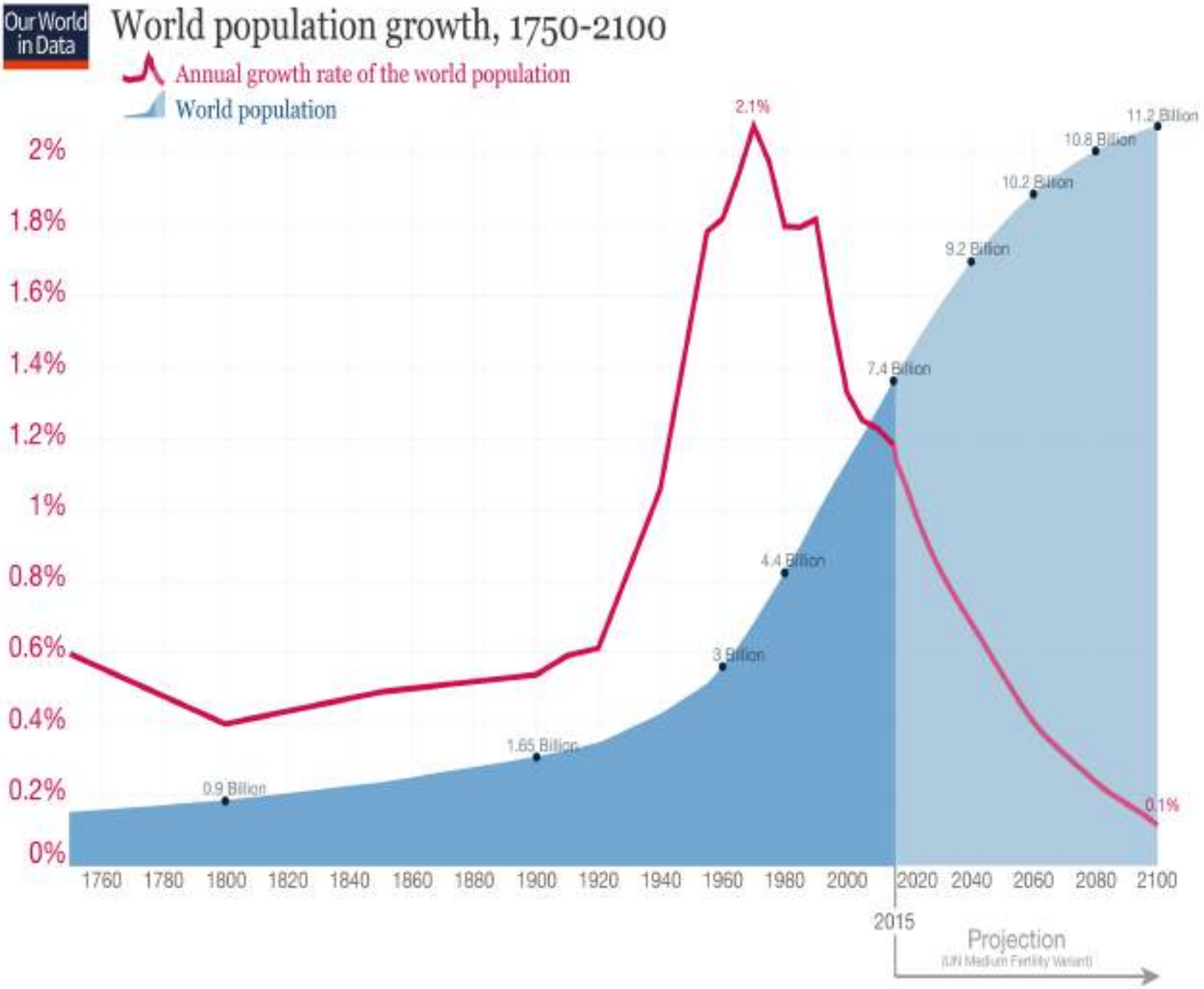
P.F. Verhulst
1804-1849

$$f(x) = 1/(0{,}3 + exp(-2x))$$





Fenomeni di crescita in biologia ed economia: ragionare sul cambiamento come educazione alla razionalità


Our World in Data
World population growth, 1750-2100
Annual growth rate of the world population
World population
2%
1.8%
1.6%
1.4%
1.2%
1%
0.8%
0.6%
0.4%
0.2%
0%
2.1%
0.1%
0.9 Billion
1.65 Billion
3 Billion
4.4 Billion
7.4 Billion
9.2 Billion
10.2 Billion
10.8 Billion
11.2 Billion
1760
1780
1800
1820
1840
1860
1880
1900
1920
1940
1960
1980
2000
2020
2040
2060
2080
2100
2015
Projection
(UN Medium Fertility Variant)
Data sources: Up to 2015 OurWorldInData series based on UN and HYDE. Projections for 2015 to 2100: UN Population Division (2015) – Medium Variant.
The data visualization is taken from OurWorldInData.org. There you find the raw data and more visualizations on this topic.
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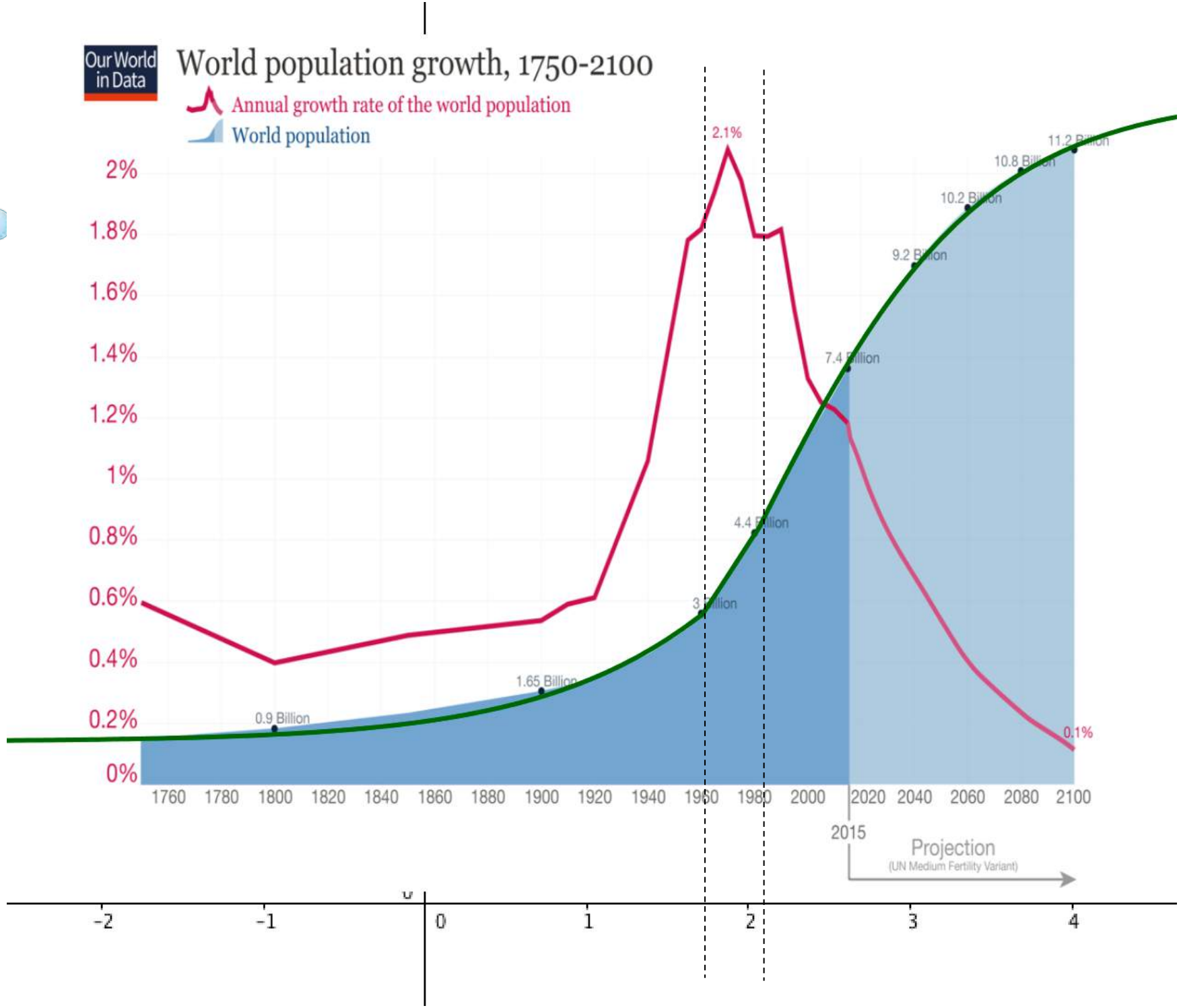
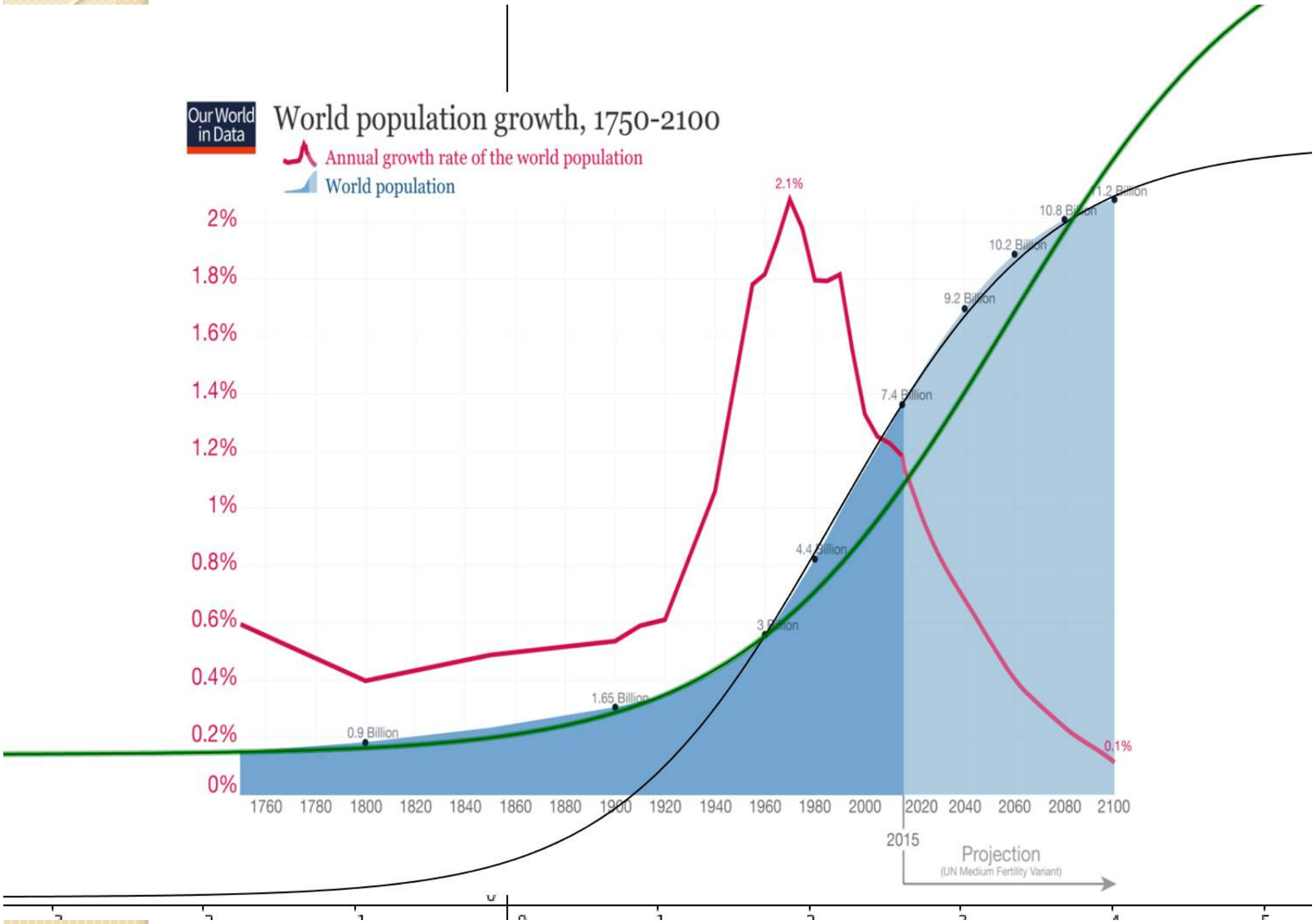
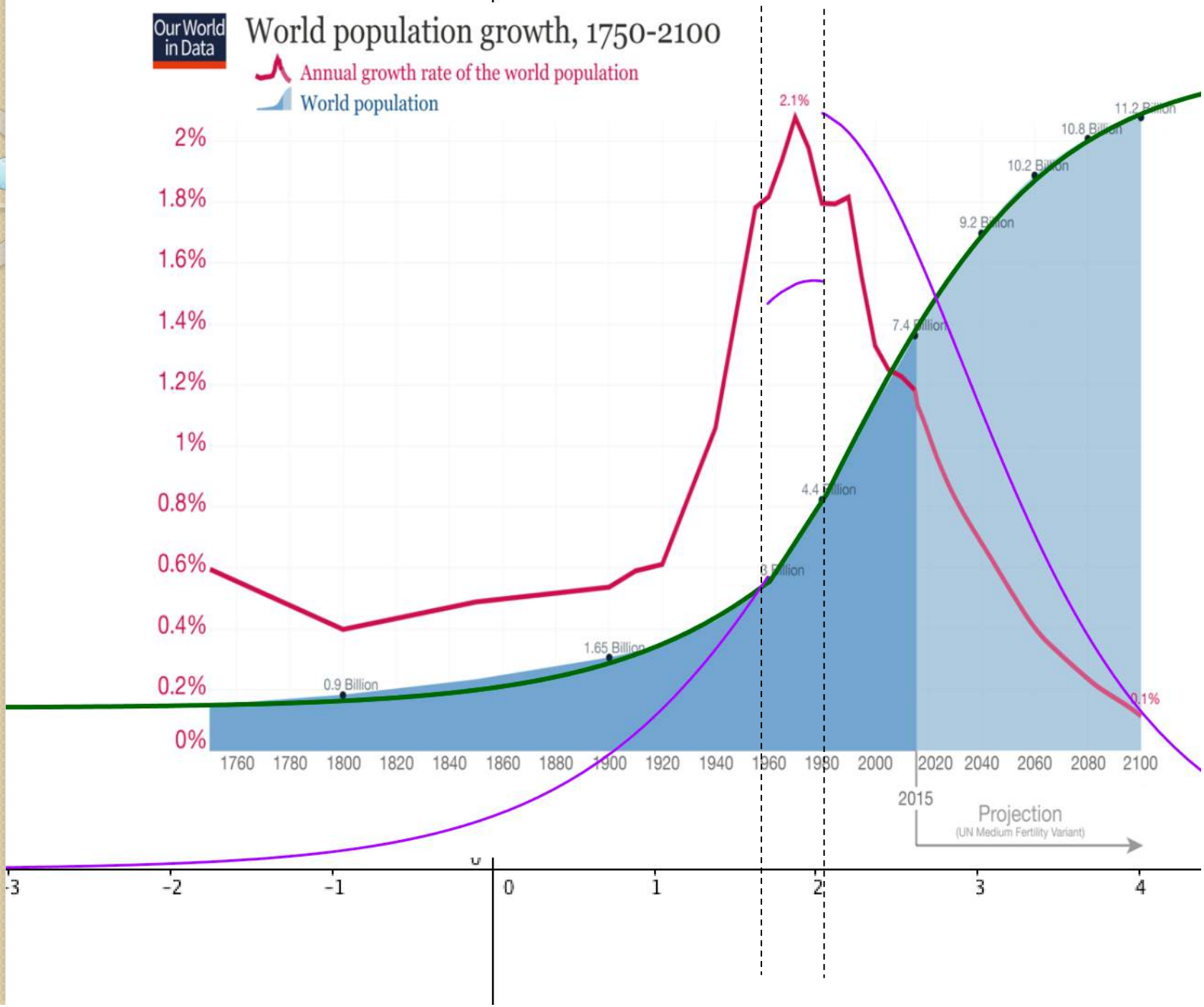

Our World in Data
World population growth, 1750-2100
Annual growth rate of the world population
World population
2%
1.8%
1.6%
1.4%
1.2%
1%
0.8%
0.6%
0.4%
0.2%
0%
2.1%
0.1%
0.9 Billion
1.65 Billion
3 Billion
4.4 Billion
7.4 Billion
9.2 Billion
10.2 Billion
10.8 Billion
11.2 Billion
1760 1780 1800 1820 1840 1860 1880 1900 1920 1940 1960 1980 2000 2020 2040 2060 2080 2100
2015
Projection
(UN Medium Fertility Variant)
-2 -1 0 1 2 3 4


Our World in Data
World population growth, 1750-2100
Annual growth rate of the world population
World population
2%
1.8%
1.6%
1.4%
1.2%
1%
0.8%
0.6%
0.4%
0.2%
0%
2.1%
0.1%
0.9 Billion
1.65 Billion
3 Billion
4.4 Billion
7.4 Billion
9.2 Billion
10.2 Billion
10.8 Billion
11.2 Billion
1760 1780 1800 1820 1840 1860 1880 1900 1920 1940 1960 1980 2000 2020 2040 2060 2080 2100
2015
Projection
(UN Medium Fertility Variant)


Our World in Data
World population growth, 1750-2100
Annual growth rate of the world population
World population
2.1%
0.1%
2%
1.8%
1.6%
1.4%
1.2%
1%
0.8%
0.6%
0.4%
0.2%
0%
0.9 Billion
1.65 Billion
4.4 Billion
7.4 Billion
9.2 Billion
10.2 Billion
10.8 Billion
11.2 Billion
1760 1780 1800 1820 1840 1860 1880 1900 1920 1940 1960 1980 2000 2020 2040 2060 2080 2100
2015
Projection
(UN Medium Fertility Variant)
-3 -2 -1 0 1 2 3 4

P.F. Verhulst
1804-1849

a = 7.8

b = 3

$V(x) = b/[1+a^{(-x)}]$

$V^*(x)=V'(x)/V(x)$

V(x)

V*(x)